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ISSUE No. 10

# PRACTICAL MATHEMATICS

THEORY AND PRACTICE WITH MILITARY AND INDUSTRIAL APPLICATIONS

**STRUCTURAL ENGINEERING**

**Properties**

*Stress and Strain*
*Deformations*
*Factor of Safety*

**Moving Bodies**

**Columns** **Beams**

**MACHINE-SHOP PRACTICE**

**Cutting Speeds**
**Time for Machining**
**Threads** **Tap Drills**
**Indexing**

—ALSO—

*Mathematical Tables and Formulas*

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Princeton University

35¢

EDITOR: REGINALD STEVENS KIMBALL ED.D.

ISSUE 10 **Practical Mathematics** VOLUME 2

REGINALD STEVENS KIMBALL, Editor



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## CHATS WITH THE EDITOR

NOW that we are embarking on the "second leg" of our mathematical journey, it seems appropriate, in this first issue devoted to applied mathematics, to sound once again a few words of caution.

In the first place, some of you, in your eagerness to "get ahead" with this part of the course, may feel inclined to jump right into the reading of this issue even though you still have some unfinished work left over from preceding issues. If you do this, don't be surprised if, at some point, you find yourself suddenly wondering what it's all about. You will have no difficulty up to a certain point, probably, but past that point things will be anything but clear. This point—a variable, depending on how far you have carried your study of theoretical mathematics—will differ for different readers.

*Caution number one*: When you reach that point, take it as a signal for some review. Turn back to your file of the earlier issues and pick up the thread at the point where you seem to have dropped it. Perhaps, in the light of the applications now being made, some of the formulas which seemed useless and stuffy will begin to take on new meaning for you.

*Caution number two*: The chances are still ten to one that most of your errors will be in a simple arithmetic. You will remember that we quoted statistics on that point in earlier chats and urged you to give particular care to building up a good foundation. Even the staff of mathematical experts whom we have employed to solve the exercises in this course have shown weaknesses in the fundamental operations. Not all of these weaknesses—in their cases or in your case—are due to ignorance; many of them are due to carelessness (or, perhaps, it would be better to say haste). With our attention focussed on the larger issues involved in a problem, we are all of us prone to overlook the trouble which an unguarded use of a partially-learned and ill-remembered set of values may cause.

It's something like the difficulties we sometimes have in using the telephone. Most of us have had the experience, at some time or other, of calling a number which is perfectly familiar to us, a number that we call day after day. On a particular occasion, for no apparent reason, we inadvertently transpose the digits that make up the number and thus get the "wrong party". That is the reason the telephone companies warn us always to consult the directory if we have any doubt whatever about the number.

That same injunction should apply in your work in mathematics. Whenever you are in doubt as to the correct value to be used, consult the proper table to get the value. If you are constantly working with certain values, you will find, after some use, that you do not need to verify familiar values every time you come to them. Most of us know that $\pi$ is $3\frac{1}{7}$ or 3.14; many of us know that it is 3.1416 or 3.14159. Few of us have occasion to use it exact to the fifteenth decimal place, and it is sheer braggadocio or time-extravagance to bother to learn its value that exactly.

By this time, you should have developed for yourself a list of the values which you are constantly needing in your work and should have made those particular values as readily comprehensible as $2 \times 2$. If there are some you feel you ought to know better than you do, jot them down on a small card, which can be carried readily in your pocket. At odd moments during the day, when riding to or from work, when adjusting your necktie or scraping the beard off your chin, or when you find an idle moment, consult the card and commit to memory a few more values.

*Caution number three*: Don't try to learn too many new values at once. Select two or three as your special project for the day; concentrate on these until you have mastered them. On the following day, review these and add two or three more. On the third day, review the whole half-dozen and add another trio. When you feel that you know any particular values well—after five or six or ten days of review in this fashion—drop them off your daily list, but plan to return to them once a week for a while thereafter. You'll surprise yourself after a while at the amount of memorized material which you have been built up.

*Caution number four*: In the same way, plan to review from time to time a whole article which you have previously studied. Maybe you're getting a little rusty on logarithms. Maybe you are finding difficulty in getting the exact readings from the slide rule which you feel you ought to be getting. Perhaps the binomial theorem is a bit hazy in your mind. It may be that some of the trigonometric formulas which you took in stride when you studied the issue are not so readily called to mind in case of need as they were when you were working on that issue. You may have forgotten the formula for finding the value of an angle of a regular polygon or for finding the volume of a solid figure. At the very moment when you find uncertainty on a point like this, resolve to take time out to go back and check yourself on this matter. That's the way to make these applied issues serve their purpose in helping you to strengthen your acquisition of the whole realm of mathematics.

Another point that I feel I ought to mention again at about this time: Are you filing your solutions in an orderly fashion, so that you may refer to them easily when you have occasion to review any particular topic? As we have suggested in the introduction to the workbook which accompanies this series, orderliness in setting up problems often helps in spotting errors which may have crept in along the way.

It may seem like a waste of time, but it's good policy, nevertheless, to check every solution. We have given you suggestions all along the way, in the theoretical issues, of various devices which may be utilized for verifying the accuracy of a solution. A check at each step in the development of a long and intricate problem may save you the labor of doing a whole problem over again if you wait until you get to the very end before making the check. Any time that you use a "wrong" figure in a step, that step is worthless; make sure, then, that the figure is correct before you incorporate it in your further calculations.

In most of the work in applied mathematics, you can save yourself a great deal of figuring if you learn to rely on the logarithm tables or the slide rule. Most of what you need to know about these subjects has been covered in the articles in Issues Two and Seven. There are many additional "wrinkles" which you can train yourself to pick up in specific

uses. Any of the slide-rule manuals or the introductory matter in the front part of books on the larger logarithmic tables will help you to amplify your use of these devices.

Until you are sure of yourself in using logs or the slide rule, it is good practice to work the solution out the "hard way", going through the writing of the multiplications and divisions, and then checking your solution by the mechanical methods. If you come out right to the indicated number of significant figures, you may be sure that you are grasping the principles in using logs or the slide rule. When you are thus assured of your proficiency, you may discard the "long, hard" way and rely on the mechanical means of saving time.

Now, a word about the present issue. We publish here two major articles: one on construction engineering and one on machine-shop practice. Perhaps these are not the subjects in which you have been particularly interested or for which you feel that you are going to have a particular need. Surprisingly enough, you may discover, as I did when the manuscripts came in, that there is a great deal in these subjects which everyone ought to know just as a matter of general knowledge.

If, then, you are, like me, merely a "consumer" of the information in this issue rather than an active worker in these fields, you will, of course, be more concerned with following through the steps by which the processes are built up than with committing to memory the formulas which would be used under given conditions. On the other hand, if you are vitally concerned with either one of these topics, you will desire to give your major attention to its development.

As Mr. Sollenberger points out in the introductory section of his article, everyone is, daily, dependent on construction engineering for his own safety and welfare. As we follow along with Mr. Sollenberger, we discover that more enters into the problem than would have seemed from casual observation, when we watched a building in process of erection, to be the case. Even the seemingly simple problem of where the painter should tie the anchorage of his scaffold takes on new implications when we find it in its context here.

Mr. Benedict's article, the fruit of years of experience in an eminently practical institution, reduces to simplest terms the mathematical principles which one needs to know for calculating the ordinary operations in a machine shop. With the present trend of many white-collar workers toward industrial jobs as a part of the war effort, we believe that this article will prove one of the most popular and beneficial in our whole series.

It will be an interesting exercise, in connection with either one of these articles, to go through and try to determine for yourself just how big a part arithmetic plays in the solution of the problems. You will be calling on the higher branches for help at various points, it is true, but every formula, at some point, resolves into a demand for a simple arithmetical calculation. (Yes, I'm harping on the same old theme, again! That's one point that I hope to get across to you before this course is over.)

We shall proceed next to a consideration of the use of mathematics in problems of heat and chemistry. In Issue Number Eleven, Dr. Hynes and Dr. Pickering will guide your thinking along these lines.

R.S.K.

## ABOUT OUR AUTHORS

DURING his entire career as student and teacher of construction engineering, Norman J. Sollenberger has devoted much time to actual field work. Between 1928 and 1932, he spent five summers at highway bridge and pavement construction labor. From February to June of 1933, he was foreman of a city paving job at Stafford, Kansas. In 1936, he was resident engineer in connection with three bridge projects for Wabaunsee County, Kansas. During the summer of 1937, he was a draftsman in the bridge design department of the Kansas State Highway Commission. Two years later, he worked in the shops of the Pittsburgh-Des Moines Steel Company. This was followed in 1940 by a period as estimator for a contracting company at Sioux City, Iowa, and a turn at concrete inspection for an engineering firm in Chicago, Illinois.

These facts are recited to convince the reader that Mr. Sollenberger's article is no routine piece of pedagogy. The editor believes that it combines magnificently the two principles of PRACTICAL MATHEMATICS, theory and practice.

Mr. Sollenberger was born at Chapman, Kansas, in 1912. He intermittently attended Kansas State College from 1930 to 1936, receiving the degree of Bachelor of Science in Civil Engineering in 1935, and the degree of Master of Science in Civil Engineering in 1936. He remained at Kansas State until 1937, teaching applied mechanics, and then accepted an appointment at Iowa State College where he taught theoretical and applied mechanics until 1941. In the summer of that year, Mr. Sollenberger came to Princeton University as an instructor of civil engineering. He became an assistant professor this spring. Mr. Sollenberger is also head of the Materials Testing Laboratories at Princeton.

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THE manuscript on machine-shop practice has been written by Otis Benedict, Jr., who is able to draw upon a great wealth of practical experience as well as theoretical knowledge. For many years, Pratt Institute has been recognized as a leader in training students in the technique of the machine shop. Mr. Benedict's long service with this institution in connection with the machine-shop courses offers its own recommendation as to his qualifications as author of this important section of PRACTICAL MATHEMATICS.

Mr. Benedict was born in 1900 at Scranton, Pennsylvania. An early predilection for his chosen profession was manifested by his decision to enter the Scranton Technical High School. After graduation, he spent the next five years gaining experience in commercial machine-shop work, specializing in tool- and die-work, jigs, and fixtures. Convinced that a thorough academic training was necessary, he then entered Pratt, from which he graduated in 1925. From Pratt, Mr. Benedict went for a year with Yale & Towns, Stamford (Connecticut) lock manufacturers. He returned to the Institute to begin a tenure as instructor which at the present writing adds up to seventeen years.

Mr. Benedict is a member of the American Society of Tool Engineers, and author of *Machine Shop Manual*, a recognized text in its field. Like most of our authors, he is now engaged in extra-curricular activity in connection with the war effort. He is serving as supervisor of the war training classes in machine-shop practice at Pratt.

## · STRUCTURAL ENGINEERING ·

By Norman J. Sollenberger, M.Sc. in C.E.

A COMPLETED structure, whether it be large or small, will impress the individual in a multitude of ways. Some regard the structure as a mere convenience and do not give it a second thought. Others will see in this object a beauty which cannot be expressed in any other way than as an excellent blend of usefulness, economy, and appearance. Only the trained engineer who is the force behind the erection of the structure can see the endless planning, the many details, and the assortment of skills that were required to produce the object we have referred to as a structure. This structure may be a towering skyscraper, a long suspension bridge, a sleek body of an airplane, or a small delightful cottage in your neighborhood.

The architect is the first man to consult if one wishes to bring a new structure into being. He is the man who suggests its size, its shape, and its color combination; and he should also suggest certain additions or corrections which would increase the convenience or economy of the resulting structure.

Next to come into the picture is the structural engineer, who takes the suggested size and shape and determines the actual size of each beam and wall required. He is the man who is well founded in mathematics and physics, for the scientific approach to this subject has proved to pay many dividends. To use a beam which is twice the size required is poor economy, and to use a beam which is half the size required is obviously suicidal. The public places its collective life in the hands of structural engineers many times each day by simply walking into a building or across a bridge.

If the structure is small, the architect may act as the structural engineer, and he may also select the beams in the structure by arbitrary rules based upon experience. To this architect, the design of a structure is an empirical procedure and not a science. For a small structure, a skillful architect may be adequate; but, for a large structure, modern knowledge demands the scientific approach to the design of structural features.

The third man to play an important part in bringing a structure

into being is the construction engineer. This man, using the instructions given him by both the architect and the structural engineer, assembles the materials, the equipment, and the men required to carry out these instructions. The coördination of these many factors requires a great deal of expert management.

It is often suggested that the man who is supervising the construction work need know very little about mathematics. We grant that he need know less mathematics than the structural engineer and that most of his mathematical calculations have already been done, but it is necessary, nevertheless, that he acquire some comprehension of the load-resisting properties of the tools with which he works (scaffolds, cranes, cables, etc.) and of the structure which he is assembling.

The steps required to produce a completed structure are as follows:

**a** Visualized by the architect.

**b** Given strength by the structural engineer.

**c** Given actual existence by the construction engineer.

The public sees nothing of the first two steps. Not until work on the foundation begins does the man on the street realize that a new structure has been undertaken. Actually, months or perhaps years have passed since this structure had its beginning, for much planning and preparation have taken place regarding the new project.

Not more than a few hundred years ago, the construction of a structure was not a scientific process but an art. Master builders had acquired the art of cutting and laying stones to build excellent structures, such as the arch, by means of long experience with other builders. Through a trial-and-error process, the art of building became very well advanced, but not until after a scientific approach was used did the great structures of the present day become possible.

Fortunately, this scientific approach has reduced the number of uncertainties considerably. We still have, however, isolated instances of great loss of life or property damage. In one instance, this may be due to structural weakness that the present scientific tools did not foretell. In another instance, the engineer, owing to ignorance or fraud, may have applied the available information incorrectly.

Structural failures must have been more numerous in the days when building was still an art, for drastic measures were taken to curb careless work in this field. Good examples of this effort may be found in the laws of some of the ancients, such as the following extract from the "Code of Khammurabi":

If a builder build a house for a man and has not made his work strong, and the house has fallen in and killed the owner of the house, then that builder shall be put to death. If it kill the son of the owner of the house,

the son of that builder shall they kill. If it kill the slave of the owner of the house, a slave equivalent to that slave to the owner of the house shall he give. If the property of the owner of the house it destroys, whatsoever it destroys he shall make good, and as regards the house he built and it fell, with his own property he shall rebuild the ruined house. If he build a house for a man and did not set his work and the walls topple over, that builder from his own money shall make that wall strong.

The present laws are much more lenient than in ancient days, but responsibility still includes injury or damage to persons or property caused by negligence on the part of the constructor or his agents. This means that, should a building (during construction) fall down owing to some negligence on the part of the constructor, he is responsible for this damage. The courts will fix the amount of damage if it has not been definitely stated in the contract between builder and owner. Should life or public property be endangered, again the constructor is liable. Many localities require construction firms to carry some type of liability insurance.

In considering the application of mathematics to construction work, we must keep in mind the close connection between the structural engineer and the construction engineer (the contractor). A contractor may build a building or a bridge, and it may collapse shortly after its completion. If the contractor has performed the erection of the structure according to the plans and specifications, he is not at fault. If the engineer who formulated the design is at fault, the structure cannot be a success regardless of the excellent workmanship that may have been applied.

Faulty materials may ruin an otherwise excellent combination of brilliant design and excellent workmanship. The application of mathematics to construction work involves the basic principles underlying the choice of materials. Only after the material has been chosen is it possible to select the size of the individual members of a structure. It is desirable first to gain a fundamental knowledge of construction materials before delving into the mathematical computations of stresses and strains.

**PROPERTIES** Before one can work intelligently with materials, he must be able to describe them. In solving this problem, descriptive terms and phrases have been invented. These terms and phrases are known as *properties*. In describing a certain construction material, all that we have to do is to list the properties for that particular material. By means of this list of properties, we can specify the material to be purchased from a producer. Thus, the manuscript known as a *specification* is born.

The construction engineer has to know the properties of the material he is using in order to be assured that he is using the specified

material as well as the correct construction procedure. Since the mathematics used in this work depends to some extent upon the properties of the material under consideration, we shall begin by defining a few of the important ones.

### *Stress and strain*

On the usual construction project, the constructor is placing pieces of material such as wood, steel, or concrete which will be subjected to the following three broad loading classifications: *tension*, *compression*, and *shear*. A rope that is subjected to the forces caused by suspending a weight is subjected to a *tensile* force. The leg of a loaded table is subjected to a *compressive* force, and the scissors which parts paper by sliding one part with respect to the other is exerting a *shearing* force on the paper.

The two terms, *unit stress* and *unit strain*, represent very important concepts.

*Unit stress* is defined as the load per unit of area imposed on the structural member. This statement may be indicated mathematically by the formula, $S=\frac{P}{A}$, in which $S$ represents the unit stress, $P$ represents the total load, and $A$ represents the cross-sectional area of the member.

*Unit strain* is defined as the change in length per unit of length. This statement may be indicated mathematically by the formula, $e=\frac{\Delta}{L}$, in which $e$ represents the unit strain, $\Delta$ represents the total change in length of gage length, and $L$ represents the gage length or the length over which deformation measurements were taken.

Notice in Fig. 1 that the arrows representing the tensile and compressive stresses are perpendicular to the plane upon which they act and notice that they are of different lengths (denoting different magnitudes). The difference in magnitude of these individual unit stresses is true because of the fact that the material in which the stresses occur is not homogeneous. We have no perfectly uniform material and therefore cannot obtain the uniform distribution of stress over the cross section that the formula, $S=\frac{P}{A}$, implies. Thus it is to be kept permanently in mind that, whenever the formula, $S=\frac{P}{A}$, is employed, the user is assuming

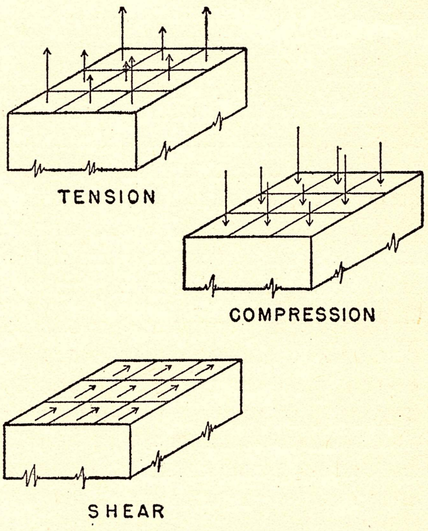



Fig. 1

a perfectly homogeneous material because the formula gives an absolutely uniform unit stress distribution.

Likewise, the change in length of a structural member may not be uniformly distributed as the formula, $e=\frac{\Delta}{L}$, implies. Therefore, a homogeneous material is again assumed.

#### TEST YOUR KNOWLEDGE OF STRESS AND STRAIN

1 A steel cable 0.25 inches in diameter is used to lift a 1,000-lb. weight. Determine the average tensile unit stress in the cable.

2 A 0.5-inch by 0.5-inch steel bar is used to suspend a one-ton load. Determine the average tensile unit stress in the bar.

3 A cube of concrete 5 inches on a side is placed under each corner of a small building. If the building weights 200 tons, determine the average compressive unit stress in the cubes.

4 A steel bar 30 inches long is subjected to a tensile force which causes the bar to elongate 0.3 of an inch. Determine the average unit strain in the bar.

### *Behavior of materials of construction*

If the exact behavior of the materials used in construction were known, much more information would be at hand than is at present available. Valuable information is being added continually to the store of present knowledge. This supply is very plentiful in some branches of the problem and very scanty in others.

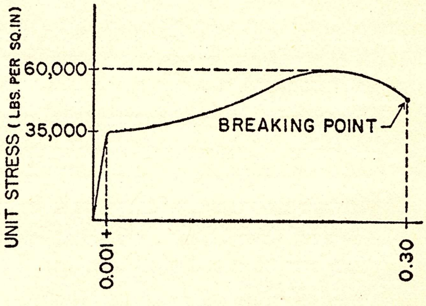
Most of us are aware of the fact that all materials are elastic to a greater or lesser degree. If a load is imposed upon a structure, some deformation is expected to accompany it. The exact magnitude of this deformation depends upon the magnitude of the unit stress and the stiffness of the material. If the unit stress is low enough, we expect the material to spring back to its original size and shape after the load is removed. Some materials will revert quickly, some slowly, and others will move toward the original size and shape but will retain some permanent deformation. Some materials have very good elastic properties and are considered elastic up to the point of fracture. Others have very good elastic properties until a certain unit stress is exceeded; then large amounts of permanent deformation will occur.

A graphical picture of the way a certain material will act as it is subjected to loading is obtained by drawing a stress-strain curve. A typical stress-strain curve for the type of steel that is used in most bridges and buildings is shown in Fig. 2. This diagram, as you see, shows the relationship between the unit stress and the unit strain for the given material.

The data for plotting the stress-strain curve is obtained by actually loading a sample of the material in tension or compression, and taking simultaneous readings of load and deformation. The total load on the specimen is changed to unit stress $\left(S=\frac{P}{A}\right)$ and the total deformation is changed to unit strain

$\left(e=\frac{\Delta}{L}\right)$. These computed results are then plotted on coördinate paper to any desirable scale.

The curve consists of a straight line for the lower stress range, which indicates that the unit deformation is proportional to the unit stress. This straight-line relationship makes computations very simple, as will be pointed out later. At the end of the straight line, you will notice a sharp change in in the slope of the curve, which becomes nearly horizontal. This indicates that a large amount of deformation is obtained with little or no increase in unit stress. Observe next that the curve rises to a maximum and then moves downward again. The maximum of the curve is known as the *tensile strength*, and indicates the maximum unit stress to which the material may be subjected before fracture can occur. The curve is terminated by the fact that the bar actually breaks in two.



**Stress-Strain Curve for Structural Steel**
**Fig. 2**

The *elastic limit* (the maximum unit stress to which a material may be subjected and still return to its original shape after removal of the load) cannot be obtained from the stress-strain curve, but experience informs us that this unit stress is nearly the unit stress at the end of the straight-line portion of the curve. Therefore, a very good substitute may be obtained by reading the *proportional limit* (the maximum unit stress to which a material may be subjected and have the deformation occur in proportion to the unit stress) from the curve. Since the proportional limit may be substituted for the elastic limit, the maximum amount of elastic deformation which may occur is represented by the unit strain value under the straight-line portion, and is known as the *elasticity* of the material. Note that this elastic deformation is very small in comparison with the total deformation, known as the *per cent elongation*, which occurs before fracture. (See Table LII.)

It is important to obtain the following properties of a material before it is used in a structure: *elastic limit*, *tensile strength*, and *ductility*. The properties which refer to the strength of the material are the elastic limit and the tensile strength. A value for the elastic limit may be obtained by reading the unit stress corresponding to the end of the straight-line portion of the curve. A value of the tensile strength may be obtained by reading the unit stress corresponding to the highest point on the curve. An indication of the ductility may be obtained by reading the unit strain corresponding to the breaking point on the curve.

### TEST YOUR KNOWLEDGE OF PROPERTIES OF MATERIALS

5 Pick a value for the following properties from the stress-strain curve of Fig. 2: elastic limit, tensile strength, elasticity, and per cent elongation.

It is important to know values for these properties because it is essential to make sure that the unit-stresses placed upon the material in the finished structure are not excessive. Also, it is important to know the ductility so that construction procedures may be determined. In working with glass, one would not plan to bend it into shape without heating, nor would one punch holes into it as freely as is customary when fabricating structural steel. Nails are used when fabricating wooden structures, rivets are used when fabricating metal structures, and a needle and thread are used when fabricating clothing. All of these different procedures are determined by the properties of the material to be fabricated.

## *Deformations*

The deformation of a structural member under loading may be a very important factor. As has been stated before, the magnitude of the deformation depends upon the unit stress imposed upon the member and the stiffness of the material.

If the unit stress does not exceed the elastic limit, the unit strain is proportional to the unit stress (assuming a straight line on the stress-strain curve up to the elastic limit). The stiffness may be evaluated by determining the ratio of the unit stress to the unit strain (the slope of the straight line). The stiffness, $E$, for structural steel is $\frac{S}{e} = 30{,}000{,}000$ p.s.i. (pounds per square inch).

For any value of the unit stress below the elastic limit, the unit strain may be easily computed by substituting into the above equation $\frac{S}{e} = 30{,}000{,}000$ p.s.i. Should the unit stress be larger than the elastic limit, it is necessary to read the unit strain corresponding to this unit stress from the stress-strain curve.

### TEST YOUR ABILITY TO COMPUTE UNIT STRESS

**6** A steel bar 5 feet long is subjected to a tensile unit stress of 15,000 p.s.i. Determine the total deformation caused by this loading.

**7** Is the steel bar in problem 4 stressed above the elastic limit?

**8** An aluminum bar 3 feet long is subjected to a tensile load which causes the bar to elongate 0.036 inches. If the area of the cross section of the bar is 0.5 square inches, determine the magnitude of the tensile load.

Deformations owing to temperature are also important. The coefficient of thermal expansion is determined by experiment and may be defined as the change in length of a material per unit of length for a one-degree Fahrenheit temperature change. If the temperature rises, the length will increase; and if the temperature falls, the length will decrease.

*Illustrative Example*

Determine the change in length of a steel girder 100 ft. long caused by a change in temperature from 10 degrees Fahrenheit to 110 degrees Fahrenheit.

Linear coefficient from Table LII is 0.0000065. Total increase in length is $100 \times 100 \times 0.0000065 = 0.065$ ft. or 0.78 in. This movement of 0.78 in. per 100 ft. due to temperature changes makes it necessary to include expansion joints in the structure.

### Factor of safety

If the tensile strength of a steel is 70,000 p.s.i., one would expect a failure to occur once the tensile stress in the structure reaches this value. If the elastic limit of this steel is 40,000 p.s.i., one would expect excessive deformations to occur when the structure is loaded to this value. Rather than run the risk of having either of these failures occur, the specifications keep the maximum unit stress low enough to provide a factor of safety. The factor of safety is always greater than 1.0 and may be defined as the ratio of the unit stress considered critical to the maximum allowable unit stress in the structure.

*Illustrative Example*

Determine the factor of safety which is present if a bar of structural steel is subjected to a tensile unit stress of 10,000 lb. per sq. in.

From Table LII: tensile strength of structural steel is 60,000 lb. per sq. in.; elastic limit of structural steel is 35,000 lb. per sq. in.

If excessive deformations are not permitted, the factor of safety is $\frac{35,000}{10,000} = 3.5$. If large deformations are permitted, the factor of safety with respect to fracture is $\frac{60,000}{10,000} = 6.0$.

## EQUILIBRIUM EQUATIONS

In making certain that the unit stresses on the material in the structure are not excessive, we must know also what stresses the structure will exert upon the material. Let us now turn our attention to the method of obtaining these stresses in some of the common structures.

The fact that a body is in equilibrium is an important bit of knowledge when the forces acting upon the body are desired. Let us review the things that are implied when the statement is made that a body is in equilibrium:

Newton's observations told him that a body at rest tends to remain at rest, and a body in motion tends to remain in motion in a straight line with a uniform velocity unless acted upon by some outside force. In other words,

if there is no outside force acting upon a body, it is stationary or moving with a uniform velocity. This means that a body which is standing still or is moving with uniform velocity is in equilibrium and this includes a very large percentage of all structures.

If it has been noticed that an object is stationary (in equilibrium), it follows that all forces acting upon this body exactly equalize each other. This means that the algebraic summation of the vertical components of the forces is equal to zero, the summation of the horizontal components is equal to zero, and the summation of the rotating effects (moments) is equal to zero.

The fact that the vertical components have a zero resultant means that no unbalanced force is present to accelerate the body in the vertical direction. Since the horizontal components have a zero resultant, there is no unbalanced force present for acceleration in the horizontal direction; and since the summation of the moments is equal to zero, there will be no unbalanced rotating effect. Hence, we have three equations at our disposal. These may be written symbolically as follows:

$$\Sigma F_x=0,\ \Sigma F_y=0,\ \Sigma M=0.$$

These may be used to solve for three unknown quantities. It is important to note that the simple observation, the body is stationary, gives us license to use the equilibrium equations to obtain values for important unknown quantities—quantities such as reactions to beams, stresses in trusses, etc., which must be known before an intelligent construction effort may be realized.

Many have heard of the equilibrium equations during elementary scientific study, but the actual writing of the equations with the correct algebraic sign attached to each term was something that was provided by the text. Instructors copied the equation from the text as a convenient starting point. There is no doubt that the origin of the equation was obvious to the instructor because he was able to visualize the body in question isolated from its supports, and forces taking the place of these supports. Let us call a body that is isolated in this manner a *free body*.

In order to visualize a *free body* clearly, we should make a complete drawing. Whether this drawing be made with compasses and straight edge, or whether it be made free-hand is immaterial as long as it is carefully done. If equilibrium equations are to be written correctly, a perfect visualization of the free body is required. Perfect visualization makes it necessary that a careful drawing be made.

If a few rules are followed, the drawing of a free body becomes very simple. The important ones are as follows:

A Cut the body completely away from all supports and sketch the body in its approximate shape.

B Show all forces that have been exerted on the body by the supports by means of arrows. If the direction or point of application is not known, assume it and be sure to place the arrow on your sketch.

C Write all known magnitudes near the appropriate arrow and name all unknown values by placing a symbol such as a letter near the arrows in question.

As soon as this is done, one is in position to count the number of unknowns in the problem under consideration and thus plan one's attack.

It has been stated that arrows are to be used to indicate the force of the removed support upon the free body. Continually keep in mind that a force is a vector quantity. To describe completely a vector quantity, we must give the magnitude, direction, and line of application. An unknown direction or line of application is just as much an unknown mathematically as an unknown magnitude. Therefore, carefully denote all known directions, magnitudes, and lines of application that are available. It will profit little to solve for a direction that is already known.

### *Illustrative Example*

Observe the sign in Fig. 3a. Though you may have inspected it carefully, the chances are that you saw only the lights. That sign was hanging above your head and is probably still there. It is stationary; it is in equilibrium; therefore, all forces acting upon the sign completely balance themselves.

In order to see what forces are acting upon the sign, let us examine a sketch of the structure as a free body (Fig. 3b). First, the structure is cut completely from its supports. Second, all forces which hold it in place are denoted by means of arrows. Third, all forces are named or the value placed near the arrows as shown. The weight of the sign, 1000 lb., is acting downward and it also acts at the center of mass of the sign. In this case, the force is known completely because the magnitude, direction, and line of application of the weight are known. The forces represented by $B_v$, $B_H$, and $T$ are not completely known. Their directions and lines of application are known

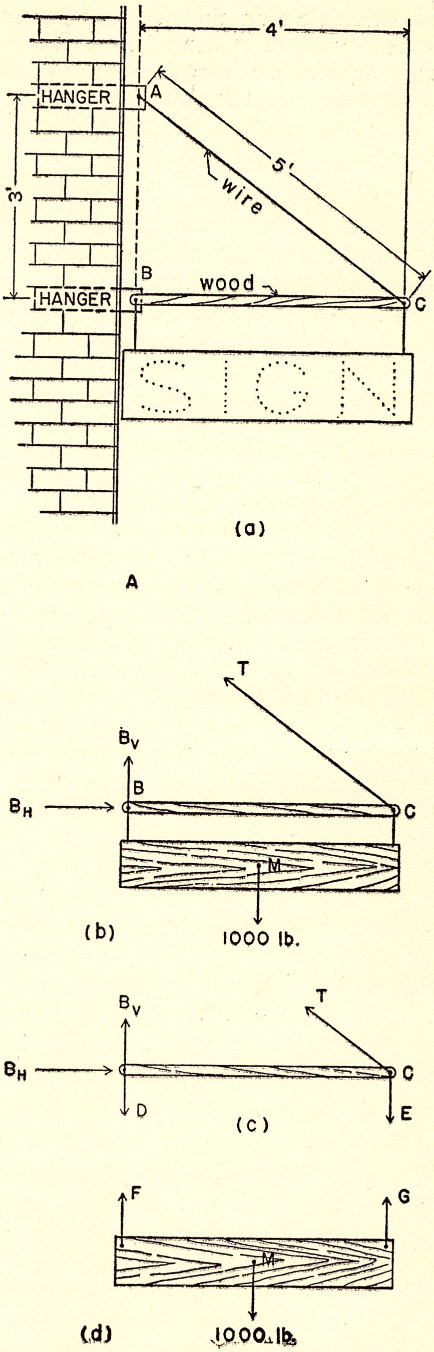



Fig. 3

and indicated on the sketch but the magnitudes are not. It is thus concluded that the magnitudes of $B_v$, $B_H$, and $T$ are the three unknowns, and three equations are required to obtain these values. Notice that the weight of all members other than that of the sign is neglected.

The rod to which the sign is hung is treated as a free body in Fig. 3c, and the sign in Fig. 3d. Notice that $D$ is opposite to $F$ and $E$ is opposite to $G$. The rod pulls upward on the sign as shown by forces $F$ and $G$. However, the sign pulls downward on the rod as denoted by forces $D$ and $E$. As for magnitude, $D$ equals $F$, and $E$ equals $G$. Force $T$ is known in direction and line of application only. $B_H$ represents the horizontal thrust that the wall exerts against the rod and is known in direction and line of application. $B_v$ represents the vertical support the wall gives the rod and is known in direction and line of application.

The forces acting upon the free body of Fig. 3c comprise a non-concurrent system with all forces acting in one plane. (See Case III of Résumé of Equilibrium Equations, page 594). Three equations of equilibrium are available and the following combination is chosen for this example:

$$\Sigma M_0=0,\ \Sigma M_i=0,\ \text{and}\ \Sigma F_x=0$$

$\Sigma$ denotes a summation.

$M_0$ refers to moments about any point on the free body.

$M_i$ refers to moments about any point other than point 0.

$F_x$ denotes forces in the horizontal direction.

The solution to the above problem is as follows:

Referring to the free body shown in Fig. 1b, let us consider the moments about point $C$. Since the structure is in equilibrium, the algebraic summation of the moments about point $C$ is equal to zero. This is written in the following equation: $\Sigma M_c=0$.

Assuming a clockwise moment to be positive, we obtain the following equation:

$$B_H\times 0+B_v\times 4-1000\times 2+T\times 0=0$$

$$B_v=\frac{2000}{4}=500 \text{ lb. upward}$$

The value obtained for $B_v$ is a positive value. This means that the arrow which represents the direction of $B_v$ was chosen in the correct direction. If the value had proved to be a negative value, this would mean that $B_v$ would be acting in the opposite direction indicated by the arrow.

Using the point, $A$, as a center for taking moments, we have $\Sigma M_A=0$; assuming a clockwise moment as positive, we obtain the following equation:

$$B_v\times 0-B_H\times 3+1000\times 2+T\times 0=0$$

$$B_H=\frac{2000}{3}=667 \text{ lb. in the direction indicated.}$$

Next, the summation of forces in the horizontal direction must equal zero.

$$\Sigma F_x=0.$$

Assuming a force acting to the right as positive, we have:

$$B_H \text{ minus the horizontal component of } T=0$$

$$B_H-\frac{4}{5}T=0$$

$$T=\frac{5}{4}B_H=\frac{5}{4}\times 667$$

$$T=833 \text{ lb. in the direction indicated.}$$

From the above analysis, we conclude the following:

**a** The tension in the wire, $AC$ (Fig. 3a), is 833 lb.
**b** The compression in the wooden member is 667 lb.
**c** Each hanger must support a vertical load of 500 lb.
**d** The top hanger must resist a horizontal tensile load of 667 lb. or it will be pulled from the side of the building.

From these computed forces, the structural designer of the sign may use the most desirable size for each member. For instance, if the allowable unit stress in the wire, $AC$, is to be 20,000 lb. per sq. in., the area of the wire should be

$$A=\frac{P}{S}=\frac{833}{20{,}000}=0.0417 \text{ sq. in.}$$

The diameter of the wire should be:

$$\pi\frac{D^2}{4}=0.0417 \text{ sq. in.}$$

$$D^2=0.0417\times\frac{4}{\pi}=0.053$$

$$D=0.23 \text{ inches.}$$

After we have explained other uses for the free body, we shall devote more space to the choosing of members for a use. The choice of the 0.23-in. wire is correct because it is subjected to a tensile load. A member in compression may buckle. This adds new complications to the problem.

***Reactions***—A *beam* or *girder* is the name given to a structural member which will resist bending. An example of such a member is shown in Fig. 4a. The forces required to support such a beam may be easily determined by means of the free body.

Since the free body (Fig. 4b) is in equilibrium, two equilibrium equations are available. Let us apply the following two equations:

$$\Sigma M_A=0 \text{ and } \Sigma M_B=0.$$

Then, as a check, determine the summation of forces in the vertical direction to see if they are equal to zero.

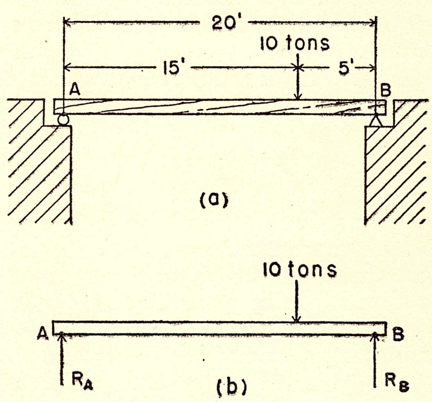


Fig. 4

*Solution*

$$\Sigma M_A=0$$

(Assume a clockwise moment as positive)

$$10\times15-R_B\times20=0$$

$R_B=7.5$ tons or 15,000 pounds upward.

$$\Sigma M_B=0$$

$$R_A\times20-10\times5=0$$

$R_A=2.5$ tons or 5,000 pounds upward.

*Check*

$\Sigma F_y=0$

(Assume an upward force as positive)

$$R_A+R_B-10=0$$
$$2.5+7.5-10=0$$
$$0\equiv0$$

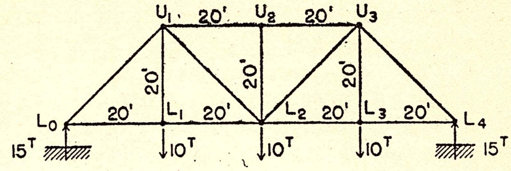


Fig. 5

***Stresses in trusses***—If the distance between supports is large (say, one or two hundred feet), it is more economical to use a *truss* to span this distance than a beam. (See Fig. 5 for sketch of a truss.) As far as determining the magnitude of the reactions is concerned, the problem is just the same as that of a beam; however, the internal stresses in a beam are different from the stresses in a truss.

As has been stated above, a beam must resist bending. A truss must resist bending also, but the first step in analyzing the stresses in a truss is to assume that all connections are hinged. This produces the condition that each individual member of which the truss is composed is subjected to pure tension or compression. The free body may be used to very good advantage when solving for the total stress in each member.

Let us now proceed with the solution of the truss in Fig. 5. As the truss is symmetrical, both reactions are equal. If a free body of the entire truss is drawn, and the equilibrium equation, $\Sigma F_y=0$, is used, each reaction is found to be 15 tons upward.

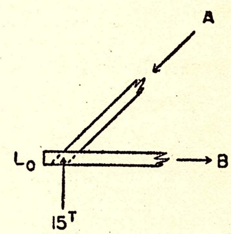


Fig. 6

Now draw a free body of the joint at $L_0$, as shown in Fig. 6. It is assumed that the members of the truss are in pure tension and compression so that the line of action of the forces, $A$ and $B$, are exactly in line with the member.

$\Sigma F_y=0$ gives the value of $\dfrac{15}{\cos 45^\circ}=21.2$ tons compression for the member

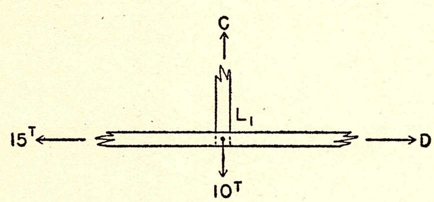


Fig. 7

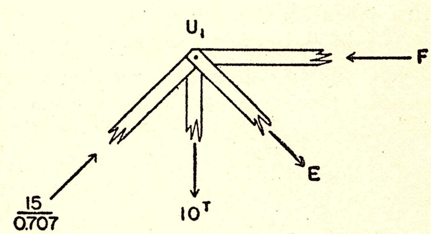


Fig. 8

$L_0U_1$, and $\Sigma F_x=0$ gives 15 tons tension in member $L_0L_1$*.

Next draw a free body of the joint at $L_1$, as shown in Fig. 7. $\Sigma F_y=0$

*Should the direction of an arrow on the free body be chosen incorrectly, the value will be found to be negative. If the arrow is changed to the correct direction, be sure to change the negative sign to a plus sign.

gives the value of 10 tons tension in member $U_1L_1$, and $\Sigma F_x=0$ gives 15 tons tension in member $L_1L_2$.

Next draw a free body of the joint at $U_1$, as shown in Fig. 8. $\Sigma F_y=0$ gives 7.07 tons tension in member $U_1L_2$, and $\Sigma F_x=0$ gives 20 tons compression in member $U_1U_2$.

This procedure may be followed until all values are determined. Knowing the total thrust on each member, you find it possible to choose the correct size.

*Illustrative Example*

The boom pole shown in Fig. 9a pivots about line $CA$, making it possible to move loads within the given radius. Determine the tension in the cable, $CB$, and the thrust in $AB$ if the weight, $W$, is 5 tons.

Draw a free body of the boom, as shown in Fig. 9b, and apply the equilibrium, $\Sigma M_A=0$. Neglect the weight of the pole.

$$5\times 8.66-C\times 5=0$$

$$C=\frac{5\times 8.66}{5}=8.66 \text{ tons (tension)}$$

Draw a free body of the connection, $B$, as shown in Fig. 9c, and apply the equilibrium equation, $\Sigma F_y=0$.

$$(\text{Vertical component of } A)-5=0$$
$$A\times \sin 30 \text{ degrees}=0.5A=5$$
$$A=10.0 \text{ tons (compression)}$$

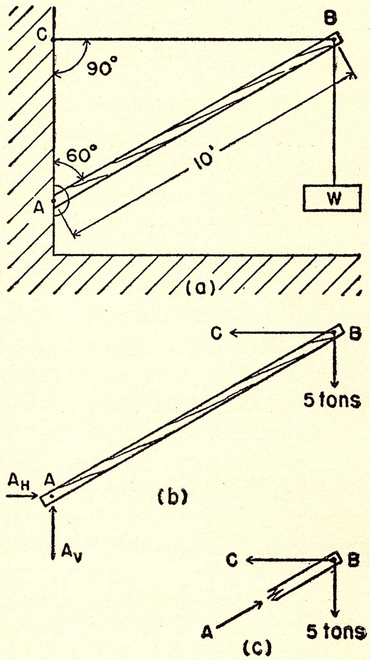


Fig. 9

**TEST YOUR ABILITY TO FIGURE STRESSES AND REACTIONS**

9 The body, $W$, in Fig. 10 weighs 1000 lbs., and all other portions may be considered weightless. Determine the tensile stresses in the cables, $AC$ and $BD$, and the tensile stresses in $CE$ and $DE$.

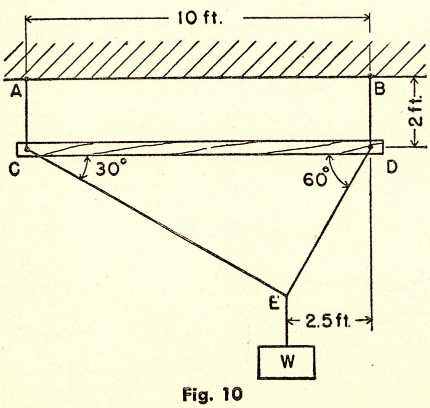


Fig. 10

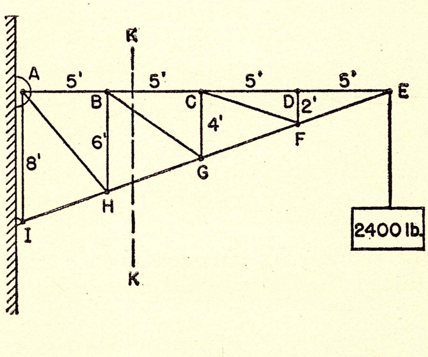


Fig. 11

**10** Determine the stresses in the member, *BC*, of the truss shown in Fig. 11. (*Hint*: Cut the truss along line *KK*, draw a free body of the portion to the right, and sum the moments about point *G*.)

**11** Determine the reactions for the beam in Fig. 12. Neglect the weight of the member.

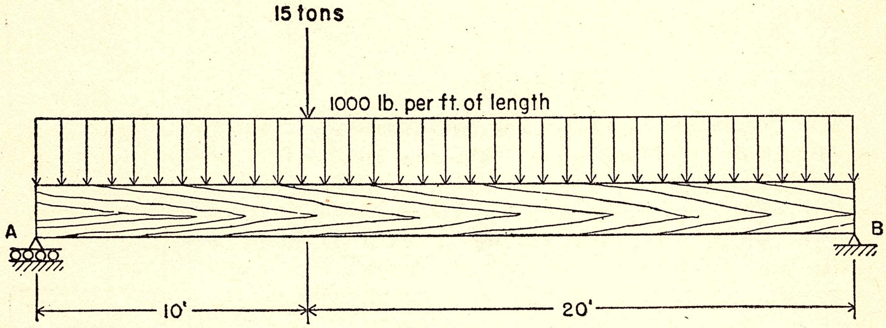



**Fig. 12**

***Ropes and pulleys***—Any type of construction work involves the use of ropes and pulleys. This is a good time to mention the subject because the free body is such an excellent tool for this type of problem.

The procedure is simply to cut all the ropes leading to and from the pulley and, at each cut section, show the force that is acting in the rope. The free body in Fig. 13b tells us that $P_1 = P_2$. This fact may be obtained by taking moments about the center of the pulley (friction is neglected). This means that should one wish to raise a weight of 100 lb. as shown in Fig. 13a, it is necessary to pull downward with an equal force. A total force of 200 lb. will result at the ceiling.

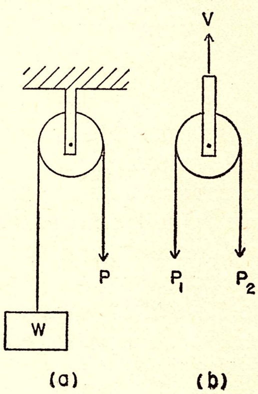



**Fig. 13**

Suppose one uses a system of pulleys as shown in Fig. 14a. Let us draw the free body shown in Fig. 14b and determine the force required to raise a 100-lb. weight. Summing the forces in the vertical direction and equating them to zero gives the following equlibrium equation:

$$\Sigma F_y = 0$$

$$100 - A - B - C - D = 0$$

Knowing that the tension in the rope is everywhere the same when friction in the pulleys is neglected, we have the equation,

$$100 = 4 \times \text{tension in rope}$$
$$\text{tension in rope} = 25 \text{ lb.}$$

This means that a downward force, $P$, slightly greater than 25 lb. will raise the 100-lb. weight. Using the upper set of pulleys as a free body tells us that the ceiling must support a total load of 125 lb. while the weight is being raised.

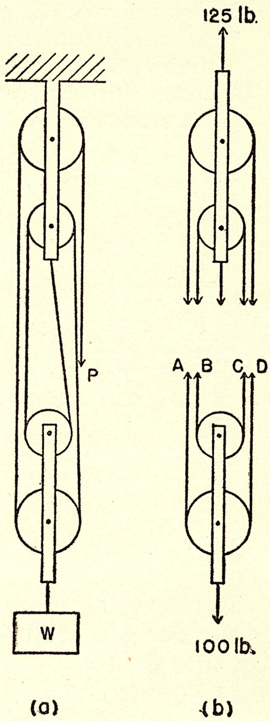



Fig. 14

#### TEST YOUR ABILITY TO FIGURE TENSION

**12** The scaffold shown in Fig. 15 is used to make it possible for workmen to paint the side of the building. The workmen simply pull the rope which is tied to the scaffold near $B$ and the scaffold will rise to a new level. The rope is again tied and the painting proceeds. If the combined weight of one workman and the scaffold is 300 lbs., what pull is required if the workman wishes to raise himself to a new level? Neglect friction in the pulleys.

**13** Assume that the support at $A$ (Fig. 15) will hold a load of 400 lb. and that, should this load be exceeded, a crash will occur. Would it be dangerous for the man on the scaffold (See Problem 12) to tie the rope to the peg at $C$ instead of to the scaffold at $B$?

***Levers***—Mechanical advantage is the only means we have of bringing large forces to bear upon an object when the available force is small. A set of pulleys is a form of mechanical advantage when they are connected so that a small force will lift a large one. A wedge is another tool which produces a large mechanical advantage. The lever is possibly the oldest means and is still very important. One is seldom conscious of the fact that a force sufficient to lift a nail from the timber in which it is driven may be developed by the human hand by means of a small lever.

In the solution of lever problems, the free body is the important item. The problem is well upon its way toward being solved if the free body is drawn and the equations of equilibrium written.

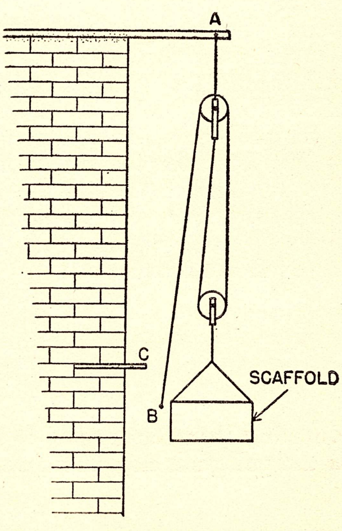



Fig. 15

*Illustrative Example*

A 6-ft. steel bar is to be used by a workman as a lever. If the distance from the load to the fulcrum is 2 inches, what load will a 200-lb. man be able to lift?

Draw the free body as shown in Fig. 16. If the load, $P$, is in the process of being raised, the steel rod is rotating about the fulcrum, $F$, with a uniform velocity. This means the body is in equilibrium and we are enabled to use the following equation: $\Sigma M_F = 0$.

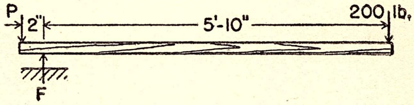


Fig. 16

$$-P\times 2+200\times 70=0$$
$$P=7{,}000 \text{ lb.} = 3.5 \text{ tons.}$$

***Thin-walled pressure containers***—Pressure containers are present in many different forms. The steam container of a steam hoist must be able to withstand a certain internal pressure. Water pipes are always subjected to some form of hydrostatic pressure. It will be worth while to analyze the stresses which these internal pressures produce in the thin shell container.

Consider the case of a water main that carries an internal pressure. Let us cut a section of this pipe one foot in length and then split it along a diameter. The result is shown in Fig. 17a. Note that the water that was in the pipe still remains in the sketch. Now show the forces which were acting upon this section before it was cut. The result is shown in Fig. 17b. The arrows show that the steel shell is in tension and that the water inside is in compression. It is perfectly logical to assume that the water will not flow out of the cut section because we have the same forces now acting that were present before the pipe was cut.

Instead of showing all of the small arrows, let us show their resultant by one large arrow acting at the centroid of the area encountered (Fig. 17c). This is the free body which is so valuable in this problem.

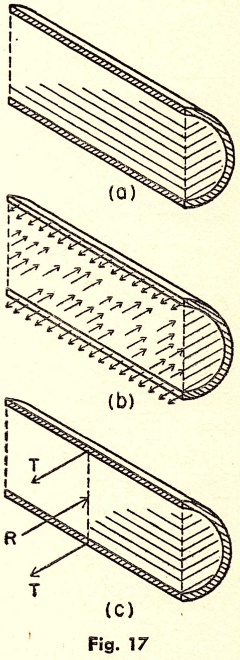


Fig. 17

*Illustrative Example*

A long cylindrical pressure container is made by welding $\frac{1}{2}$-in. steel plates together. If the inside diameter of the container is 4.0 feet, and if the allowable tensile unit stress in the steel and welded sections is 14,000 p.s.i., what unit pressure, $p$, will the container withstand?

Draw the free body shown in Fig. 18. This portion of the pipe will be stationary if no failure occurs; therefore, it is in equilibrium. Using

the equation, $\Sigma F_x=0$, where $x$ represents the horizontal direction, we have:

$$Q-2P=0$$

$$p\times 12\times 48-2\times 14{,}000\times 12\times 0.5=0$$

$$p=292 \text{ p.s.i.} = \text{allowable internal pressure}$$

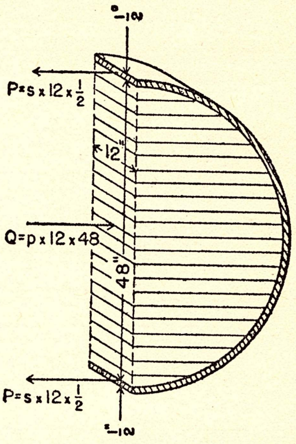



Fig. 18

***Résumé of equilibrium equations***—The beginner may be confused when he finds it necessary to choose which set of equilibrium equations to use when analyzing a problem. The available equations for three types of force combinations which may be acting upon the free body are listed below. For other combinations, see textbooks in elementary mechanics.

*Case I*—Concurrent Forces in a Plane

All forces on the free body lie in one plane and intersect at a point as shown in Fig. 19. Two unknowns may be determined from this free body and the equations are:

$$\Sigma F_x=0 \text{ and } \Sigma F_y=0$$

(Note that the $x$ and $y$ directions are not necessarily horizontal and vertical.)

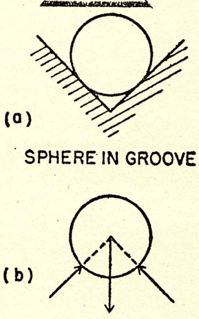



Fig. 19

*Case II*—Parallel Forces in a Plane

All forces on the free body lie in one plane and are parallel as shown in Fig. 20. Two unknowns may be determined from this free body and a choice is possible between two combinations of equilibrium equations.

First combination: $\Sigma M_0=0$ and $\Sigma M_i=0$

Second combination: $\Sigma M_0=0$ and $\Sigma F_y=0$

*Case III*—Non-Concurrent Forces in a Plane

All forces on the free body lie in a plane and do not meet at a point (See Fig. 21). Three unknowns may be determined from this free body and a choice is possible between three or more combinations of equilibrium equations.

First combination: $\Sigma M_0=0$, $\Sigma M_i=0$, and $\Sigma F_y=0$

Second combination: $\Sigma M_0=0$, $\Sigma M_i=0$, and $\Sigma F_x=0$

Third combination: $\Sigma M_0=0$, $\Sigma F_y=0$, and $\Sigma F_x=0$

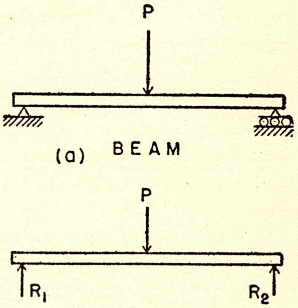



Fig. 20

**MOVING BODIES**

The free body may be used as conveniently for bodies which are in motion as for those which are stationary. Stationary bodies, as has been stated before, are in equilibrium. Therefore, the algebraic summation of the forces in any one direction is equal to zero. It follows that a body with an unbalanced

force acting upon it in any one direction will have an acceleration in that same direction. The relationship between the acceleration and the unbalanced force is very well known, $F=Ma$, in which $F$ equals the unbalanced force, $M$ is the mass of the body, and $a$ is the acceleration.

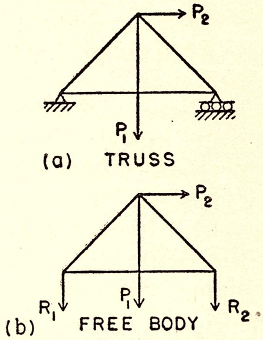


Fig. 21

Let us now consider a body that is accelerating in one given direction. If this body is isolated from its supporting forces, and all the forces are shown acting upon the body (draw a complete free body), it will be noticed that the algebraic summation of the forces in the direction of the acceleration is not equal to zero but equal to the unbalanced force which is causing the acceleration. This leads to the equation,

$$\Sigma F_x = Ma_x.$$

The subscript, $x$, denotes a given direction. If the unbalanced force is in the $x$ direction, the acceleration is in the $x$ direction also.

*Illustrative Example*

A truck is traveling on a horizontal pavement at the rate of 34.1 miles per hour when the driver suddenly applies the brakes so that the truck will slow down at a uniform rate and come to a stop in a distance of 125 feet. Determine the total friction force which must be developed between the tires and the pavement, if the total weight of the truck is 16.1 tons.

34.1 miles per hour equals 50 ft. per sec. If the truck is to stop in 125 feet, the time required is 5 seconds. The negative acceleration is 10 ft. per sec.². From the free body in Fig. 22,

$$\Sigma F_x = Ma_x.$$

$$F_1+F_2=\frac{W}{g}a_x=16.1\times 2{,}000\times\frac{10}{32.2}.$$

$$F=F_1+F_2=10{,}000 \text{ pounds.}$$

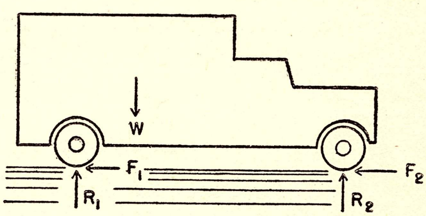


Fig. 22

Total friction force on all four tires is 10,000 pounds. This large force acting constantly through the 5 seconds gives one an idea of the difficulties involved in stopping such a large mass so quickly.

It is important to note that, as soon as a body exhibits a tendency to rotate, the inertia of rotation must be considered. No space is available at this point for explanation of this effect. See any elementary textbooks on mechanics.

**COLUMNS** Individual members of a truss may be either in tension or in compression. If the member is in tension, the area of the cross section may be determined by using the formula, $A=\frac{P}{S}$, and the size of the member required is obtained. If the member is in

compression, it must be treated as a column, which means the buckling effect must be considered.

The column problem may be divided into three parts: (a) short compression blocks where the buckling effect is negligible, (b) members which are just slender enough to make it necessary to consider both compression and bending, (c) slender compression members which bend before any large compression load can be developed. This problem does not lend itself to a simple exact mathematical solution, so empirical formulas have been devised to aid in choosing the correct size of column for a particular situation.

The load a column can carry is a function of the area of the cross section, its slenderness ratio, and the stiffness of the material. The slenderness ratio is defined as the ratio of the length of the column to the least radius of gyration $\left(\frac{L}{k}\text{, in which } k=\sqrt{\frac{I}{A}} \text{ and } L \text{ is the length of the column}\right)$ if a rolled steel section is used; or, the ratio of the length of the column to the least lateral dimension $\left(\frac{L}{d}\right)$ if a rectangular cross section is used. The stiffness of the material is the modulus of elasticity, $E$, and is the ratio of the unit stress to the unit strain $\left(E=\frac{S}{e}\right)$. The value of the modulus of elasticity is equal to the slope of the straight-line portion on the stress-strain curve.

If a *wooden* column is to be used, the empirical formula, $P=A\left(1100-20\frac{L}{d}\right)$, is often used for a good grade of wood when the slenderness ratio, $\frac{L}{d}$, is greater than 10 and less than 25. If $\frac{L}{d}$ is less than 10, the member is considered a short compression block and the formula, $P=AS$, may be used. If $\frac{L}{d}$ is greater than 25, the formula, $P=\dfrac{A\pi E}{36\left(\frac{L}{d}\right)^2}$, may be used.

$P$ = The total allowable load on the column.
$A$ = Area of cross section.
$E$ = 1,400,000 lb. per sq. in.
$L$ = Length of column.
$d$ = Least dimension on the rectangular cross section.

This and the following column formula have the safety factor included in the formula.

The American Institute of Steel Construction recommends the following formulae for *steel* columns:

$$\frac{P}{A}=17{,}000-0.485\left(\frac{L}{k}\right)^2;\ \frac{L}{k} \text{ varies from 0 to 120}$$

and

$$\frac{P}{A}=\frac{18{,}000}{1+\frac{1}{18{,}000}\left(\frac{L}{k}\right)^2};\ \frac{L}{k} \text{ varies from 120 to 200.}$$

The Institute does not recommend the use of a column with a slenderness ratio greater than 120 for main structural members; and for secondary members the slenderness ratio should not be greater than 200.

### *Illustrative Example*

A wooden column 10 ft. long is used to support a load of 20 tons. What size of timber is required if a good grade of wood is to be used?

The formula recommended for a good grade of wood is $P=A\left(1100-20\frac{L}{d}\right)$, and let us choose to use a column with a square cross section because it is foolish to provide more resistance to buckling in one direction than another unless some special loading is present on the column.

Substituting into the above formula, we have:

$$20\times2{,}000=d^2\left(1100-\frac{20\times10\times12}{d}\right)$$

$$11d^2-24d-400=0$$

$$d=7.2 \text{ in.}$$

Use an 8 in. by 8 in. wooden column, which is the next larger standard size. The $\frac{L}{d}$ ratio for the chosen column is $\frac{120}{8}=15$. This falls between the values, 10 and 25, which means that the limitations on the above formula have been satisfied.

**TEST YOUR KNOWLEDGE OF COLUMN LOAD**

**14** In the above illustrative example, a larger column than required is chosen because it is the next larger standard size. Determine the load the 8 in. by 8 in. column will carry.

**15** A 10 in. by 10 in. wide-flange I-beam weighing 60 lb. per foot is used as a column 22 ft. in length. A handbook gives the following properties for the cross section of the beam: area = 17.66 sq. in., least radius of gyration = 2.57 sq. in. Determine the maximum load this column should be allowed to carry.

**16** If the I-beam used in Problem 15 is used as a column 30 ft. in length, determine the maximum load this column should be allowed to carry.

**BEAMS** The resistance of a beam to bending loads may be determined mathematically with only a few assumptions.

This formula, which may be rigorously derived, is $M=S\frac{I}{c}$, in which $M$ is the bending moment in the beam, $S$ is the unit stress on the outside fiber of the beam, $c$ is the distance from the centroid of the cross section to the outside fiber, and $I$ is the moment of inertia of the area of the cross section about the centroid. $\frac{I}{c}$ is often referred to as the section modulus. If the beam in question has a rectangular cross section, the section modulus may be readily computed, but if the beam is a standard rolled I-beam, the evaluation of the section modulus becomes difficult. For this reason, handbooks are published which furnish this information for all standard sections.

# · MACHINE-SHOP PRACTICE ·

**By Otis Benedict, Jr.**

MANY war-industries workers, who had previously considered that they did not need much mathematics because they were doing routine work, are discovering that the man with an ability to adapt mathematical principles to his work is of more value on the job. It is our aim in this article to assist readers of PRACTICAL MATHEMATICS to make the necessary adjustments in the course of their daily work.

## CUTTING SPEEDS

Cutting tools are essentially power-driven wedges forced into the metal to remove surplus material. Whether the cutting takes place in the lathe when turning or boring, or in the drill press when drilling, or in the milling machine when milling, the cutting action is much the same.

In the removal of the surplus material by the cutting tool, heat is generated. Some of this heat passes into the metal, and some of it is absorbed by the cutting tool.

If the speed (revolutions per minute) of the cutter or the metal being cut is such that, in the removal of the surplus material, the heat generated causes the cutting tool to lose its hardness, it ceases to cut and may be made useless as a cutting tool thereafter. Many cutting tools are ruined by being operated at too high a speed.

When metal is cut in a lathe arranged to vary the speed 2 or 3 revolutions per minute at a time, the speed can be increased until the heat generated in cutting will cause the tool to lose its hardness, and cutting will cease.

If the diameter of the metal (work) being cut and its speed (rpm) are recorded when the cutting tool loses its hardness, the cutting speed at which the tool failed to cut can be calculated.

In practice, the cutting tool would be operated at a cutting speed lower than the cutting speed at which the tool failed.

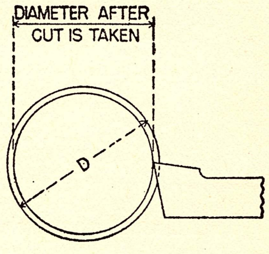


Fig. 23

The rate at which a tool passes over the work is known as the cutting speed. It is the distance in feet which the tool point cuts in a minute. If a piece of work is turned in the lathe (Fig. 23) and the point of the lathe tool cuts 20 feet (measured around the circumference of the work) in one minute, the cutting speed is said to be 20 feet per minute. (See Table LIII.)

On the planer and shaper, the cutting speed is the length of cut that would be taken in one minute. If 20 seconds $\left(\text{or } \frac{1}{3} \text{ minute}\right)$ is required to take a cut 18 feet long, in one minute the cut would be $3 \times 18$, or 54 feet. The length of cut in one minute is 54 feet and the cutting speed is said to be 54 feet per minute.

When a hole is being drilled in the drill press, the cutting speed is the number of feet travelled by the outer corners of the cutting edges in one minute.

### *Lathe, boring mill, and drill press*

The problems in cutting speeds for the lathe or boring mill can be divided into two groups:

**a** *To find the cutting speed*—The number of revolutions which the work makes in a lathe or boring mill, and the diameter are known. What is the cutting speed?

A brass rod $1\frac{1}{2}$ inches in diameter is being turned. By counting the number of revolutions of the spindle of the lathe by means of the speed indicator

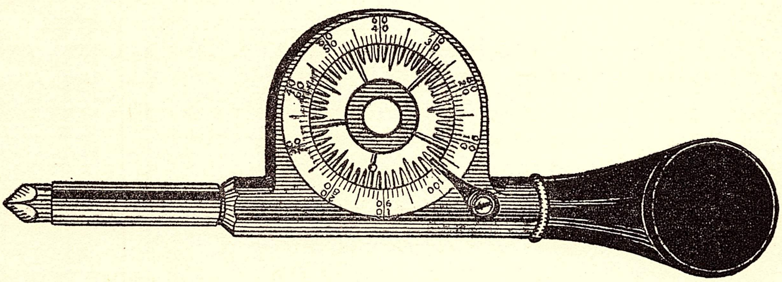


Fig. 24

(Fig. 24) we find that the work revolves 252 revolutions per minute. To find the cutting speed, we first compute the circumference of the work and change it to feet. (The circumference in inches is equal to the diameter in inches $\times$ 3.1416.) Therefore, the circumference of the work in this case is $1.5 \times 3.1416 = 4.712$, and $4.712 \div 12 = 0.393$, the circumference in feet, or the distance passed over by the point of the tool for each revolution. During 252 revolutions, the distance passed over by the point of the tool is $252 \times 0.393 = 99.04$ feet, which is the cutting speed in feet per minute.

The formula for this calculation is written:

$$\begin{array}{l}\text{cutting speed in}\\ \text{feet per minute}\end{array} = \frac{\text{diam. of work in inches} \times 3.1416}{12} \times \text{rpm} \qquad \text{I}$$

If $N$=number of revolutions per minute (rpm), $S$=cutting speed in feet per minute, and $D$=diameter of work in inches, this formula can be written:

$$S=\frac{D\times 3.1416}{12}\times N \qquad \text{II}$$

If, in this formula, $D$=diameter of the work, or diameter of bored or drilled hole in inches; the formula can be used for finding the cutting speed of drills and boring tools.

When the cut taken on a piece of work being turned or bored is deep in proportion to the diameter of the work, it is preferable in calculations for the cutting speed and revolutions per minute to consider the mean diameter being cut instead of the outside diameter of the work, and use the value for the mean diameter in the calculations and formulas given. When the outside diameter and the depth of the cut are known, the mean diameter equals the outside diameter minus the depth of cut.

**b** *To find the revolutions per minute (rpm) of the cutter*—The diameter of the work turned in a lathe (Fig. 23) or in the boring mill and the required cutting speed are known. How many revolutions per minute should the work make?

Assume that the diameter of the work is 4 inches, and a cutting speed of 40 feet per minute is required. Find the speed (rpm) of the work. Since the diameter of the work is known, its circumference equals the diameter×3.1416. Therefore, the circumference of the work is 4×3.1416=12.566 inches (or 12.6 inches is near enough for calculations of this kind). For each revolution of the work, the length of its circumference passes the tool point once; thus, a length of 12.6 inches passes the tool for each revolution. As the cutting speed is always expressed in feet, the length (12.6 inches) should also be expressed in feet. This is done by dividing by 12; thus, 12.6÷12=1.05 feet as the circumference of the work. Now the question is, "How many revolutions, each equal to 1.05 feet, does it require to get a cutting speed of 40 feet?" The answer is obtained by dividing 40 by 1.05. The result of this division is 38.04, and 38.04 is the required number of revolutions per minute to obtain a cutting speed of 40 feet per minute. In practice, 38 revolutions would be used.

The formula for this calculation is written:

$$\text{rpm}=\frac{\text{cutting speed in feet per minute}}{(\text{diam. of work in inches}\times 3.1416)\div 12}. \qquad \text{III}$$

Using the same letters to denote the quantities in this formula as in II, the formula can be written:

$$N=\frac{S}{(D\times 3.1416)\div 12}=\frac{12\times S}{D\times 3.1416}. \qquad \text{IV}$$

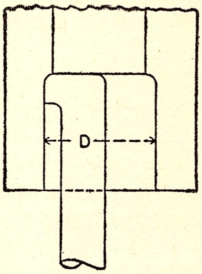


Fig. 25

If it is required to bore a hole 4 inches in diameter (Fig. 25), instead of turning a piece of work 4 inches in diameter, and the same cutting speed of 40 feet per minute is required, the calculation for the revolutions per minute is carried out in the same manner as above. Formulas III and IV are used, except that in the formulas we write "diameter of the hole in inches" instead of "diameter of the work in inches."

Formulas III and IV can also be used for work done in the drill press by substituting "diameter of the hole to be drilled in inches" for "diameter of work in inches".

Thus, if $D$ = the diameter of the work to be turned, or the diameter of the hole to be drilled or bored in inches, then formulas III and IV apply to turned, bored, or drilled work.

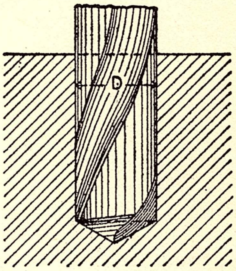



Fig. 26

*Illustrative Problem*

Find the rpm to turn a $4\frac{1}{2}$-inch diameter cast iron pulley in the lathe, when a high-speed steel cutting tool is used.

*Solution*

The cutting speed from Table LIII is 50 feet per minute. (Soft cast iron and H. S. steel)

$$N = \frac{12 \times S}{D \times 3.1416} \text{ (Formula IV)}$$

$$N = \frac{12 \times 50}{4.5 \times 3.1416} = 42.4 \text{ rpm.}$$

**TEST YOUR KNOWLEDGE OF LATHES AND DRILL PRESSES**

1 Find the rpm to turn a 14-inch diameter cast iron pulley in a lathe, when a high-speed steel tool is used.

2 Calculate the cutting speed in turning a brass rod $1\frac{3}{4}$ in. in diameter at 152 rpm.

## *Milling cutters*

In the milling machine, the milling cutter is mounted on an arbor and rotates. The cutting speeds of milling cutters can be found when the diameter of the cutter and the revolutions per minute are given.

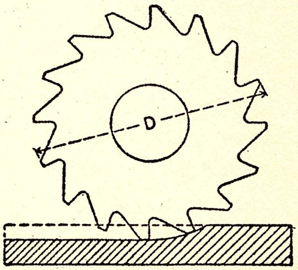



Fig. 27

*Illustrative Problem*

It is required to find the cutting speed in feet per minute, when the diameter of the cutter is 5 inches and it makes 40 revolutions per minute.

To find the cutting speed in feet per minute, first find the circumference of the cutter; thus, $5 \times 3.1416 = 15.708$ or 15.7 inches. Change this to feet; thus, $15.7 \div 12 = 1.308$ ft. Since the cutter makes 40 revolutions per minute, the cutting speed is $40 \times$ the circumference, or $40 \times 1.308 = 52.3$ feet per minute.

If, in formula II, $D$ = diameter of cutter, this formula can be used to find the cutting speed of milling cutters.

If the cutting speed of a cutter is given and its diameter known, the number of revolutions per minute at which it should be run can be found by formula IV. In this case, $D$ is the diameter of the milling cutter in inches.

**TEST YOUR KNOWLEDGE OF MILLING CUTTERS**

3 Find the cutting speed for a milling cutter 8 inches in diameter that makes 30 rpm.

4 A high-speed steel cutter 7 inches in diameter is used to take a heavy cut in a piece of cast iron in the milling machine. What should be the rpm of the cutter?

## *The crank shaper*

In a crank shaper, the tool is given a reciprocating motion. This reciprocating motion is produced by a crank arrangement built into the shaper.

In Fig. 28, the path of the crank pin is represented by the circle. The arc, $C$, represents the drive for the forward or cutting stroke of the cutting tool, and arc $R$ represents the return stroke. Find the cutting speed in feet per minute, when the length of the stroke in inches and the number of strokes per minute are known.

For example, the length of the stroke is 12 inches, and the number of strokes per minute is 60. When the cutting tool travels the distance of 12 inches, the crank pin will have travelled the arc, $C$, or $\frac{3}{5}$ of a revolution. In $\frac{1}{5}$ of a revolution of the crank pin, the cutting tool will travel 4 inches, and in $\frac{5}{5}$ (or one revolution) the tool will travel $5\times4$ inches, or 20 inches.

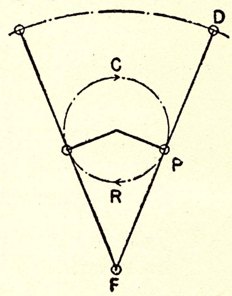


Fig. 28

From Fig. 28, we know that the tool travels 12 inches to one revolution of the crank pin, but the tool would travel 20 inches if it moved forward only, when the crank pin makes one revolution. This same result can be obtained by multiplying the length of the stroke by the reciprocal of the part of a revolution of the crank pin required to produce the forward movement of the tool; thus, $\frac{5}{3}\times12=20$ inches. Change this to feet; thus, $20\div12=\frac{5}{3}$ feet. Since the cutting tool makes 60 strokes per minute, the cutting speed is $60\times$the number of feet, or $60\times\frac{5}{3}=100$ feet.

This calculation is expressed by the formula:

$$\begin{matrix}\text{cutting speed in}\\\text{feet per minute}\end{matrix}=\frac{\begin{matrix}\text{length of stroke}\\\text{in inches}\end{matrix}}{12}\times\begin{matrix}\text{number of strokes}\\\text{per minute}\end{matrix}\times\frac{5}{3}. \qquad \text{V}$$

If $S=$cutting speed in feet per minute, $L=$length of stroke in inches, and $N=$number of strokes per minute, this formula can be written:

$$S=\frac{L}{12}\times N\times\frac{5}{3}. \qquad \text{VI}$$

Find the number of strokes per minute, when the cutting speed in feet per minute and the length of stroke in inches are known.

The cutting speed is 50 feet per minute, and the length of the stroke is 6 inches. Divide the length of the stroke in inches by 12 to change to feet; thus, $6 \div 12 = \frac{1}{2}$ foot. The length of the stroke in feet times the fraction, $\frac{5}{3}$, will give the feet cut in one stroke; thus, $\frac{1}{2} \times \frac{5}{3}$ will give the feet cut in one stroke, thus, $\frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$ foot. The number of strokes per minute is found by dividing the cutting speed in feet per minute by the feet cut in one stroke; thus, $50 \div \frac{5}{6} = 60$ strokes.

This calculation is expressed by the formula:

$$\text{number of strokes} = \frac{\text{cutting speed in feet per minute}}{\frac{\text{length of stroke in inches}}{12} \times \frac{5}{3}} \qquad \text{VII}$$

Using the same letters to denote the quantities in this formula as in formula V, we may write:

$$N = \frac{S}{\frac{L}{12} \times \frac{5}{3}} \qquad \text{VIII}$$

#### TEST YOUR KNOWLEDGE OF THE CRANK SHAPER

**5** What is the cutting speed of a shaper making 60 strokes a minute, when the length of a single stroke is 6 inches?

**6** The length of the cutting stroke in a shaper is 14 inches and the cutting speed is 80 feet per minute. Find the number of strokes per minute.

### *The planer*

In the planer, the platen (table) is given a reciprocating motion. The speed at which the platen returns when the cutting stroke is completed is usually two or more times the cutting speed. When the return speed is twice as fast as the cutting speed, the ratio of the return speed to cutting speed is said to be "2 to 1". When the return speed is three times as fast as the cutting speed, the ratio between the speeds is "3 to 1". Usually these ratios are designated "2", "3", etc. If the return speed is 80 feet and the cutting speed 40 feet per minute, the ratio is 2, while, if the return speed is 120 feet per minute, the ratio is 3.

When the number of cutting strokes per minute, the length of the stroke, and the ratio between the cutting and return speeds are known, the cutting speed and the return speed can be found. The number of strokes per minute can be counted and the length of the stroke can be measured. For long strokes, the time required for the forward stroke and the return stroke can be determined with a stop clock. The ratio of the return speed to the cutting

speed is determined by the design of the planer. Thus, the return speed and cutting speed of a planer can be readily determined.

*Illustrative Example*

The number of strokes is 6, the length of the stroke is 3 feet, and the ratio of the return speed to the cutting speed is "2". The time required for the platen to make one complete stroke (cutting stroke plus return stroke) is $\frac{1}{6}$ minute. In $\frac{1}{6}$ minute, the platen travels 3 feet in cutting and 3 feet in returning. Its rate of travel in cutting is not 3 feet in $\frac{1}{6}$ minute, but is greater than 3 feet per minute. If no return stroke were to take place, the platen would travel 3 feet plus $\frac{1}{2}\times3$ feet, or a total of $3+\frac{3}{2}=4\frac{1}{2}$ feet. (Return stroke takes place in $\frac{1}{2}$ time of cutting stroke.) The true cutting speed for one cutting stroke is then $4\frac{1}{2}$ feet and not 3 feet (the travel of the platen). The number of strokes per minute of the platen times the true cutting speed per stroke equals the cutting speed of the planer in feet per minute; thus, $6\times4\frac{1}{2}=27$ feet per minute cutting speed.

The formula expressing this calculation is:

$$\begin{array}{l}\text{cutting speed}\\\text{in feet per}\\\text{minute}\end{array} = \begin{array}{l}\text{number of}\\\text{strokes per}\\\text{minute}\end{array} + \left(\begin{array}{l}\text{length of}\\\text{stroke in}\\\text{feet}\end{array} + \frac{\text{length of stroke}}{\begin{array}{c}\text{ratio return speed}\\\text{to cutting speed}\end{array}}\right). \qquad \text{IX}$$

If $S$ = cutting speed in feet per minute, $N$ = number of strokes per minute, $L$ = length of stroke in feet, and $P$ = ratio of the return speed to the cutting speed, this formula can be written:

$$S=N\times\left(L+\frac{L}{P}\right). \qquad \text{X}$$

The return speed can be calculated by multiplying the cutting speed in feet per minute by the ratio of the return speed to the cutting speed. The cutting speed from above is 27 feet per minute and the ratio is 2. Thus, $27\times2=54$ feet per minute return speed.

If $R$ = return speed in feet per minute, this formula can be written:

$$R=S\times P. \qquad \text{XI}$$

In these formulas, the time lost in reversing is not considered.

The number of strokes per minute can be found when the cutting speed, return speed, and length of the stroke are known.

*Illustrative Problem*

Given the cutting speed of a planer as 27 feet per minute, the length of stroke 3 feet, and the ratio of the return speed to the cutting speed as "2", find the number of strokes per minute.

This problem can be solved in two ways:

**A** If the formula for cutting speed, $S=N\times\left(L+\frac{L}{P}\right)$, is solved for $N$,

we have $N=\dfrac{S}{L+\dfrac{L}{P}}$. Substituting the values from above in this formula,

we get $N=\dfrac{27}{3+\dfrac{3}{2}}=\dfrac{27}{4\frac{1}{2}}=6$ strokes per minute.

**B** The ratio between the return speed and the cutting is "2"; therefore, the return speed is twice the cutting speed, or 54 feet per minute.

As the length of the stroke is 3 feet and the cutting speed is 27 feet per minute, the time necessary to complete one forward stroke is equal to $3\div 27=\frac{1}{9}$ minute.

In a like manner, the time for a return stroke would be $3\div 54=\frac{1}{18}$ minute.

The time required for one complete stroke (forward stroke plus return stroke), therefore, is $\frac{1}{9}+\frac{1}{18}=\frac{3}{18}$, or $\frac{1}{6}$ minute. The number of strokes per minute is obtained by finding how many times $\frac{1}{6}$ is contained in one minute, or by dividing 1 by $\frac{1}{6}$; thus, $1\div\frac{1}{6}=6$, the number of strokes per minute.

In this calculation, the time lost at the moment of reversal is not considered, and a formula for the above calculation can be written as follows:

$$N=\frac{1}{\dfrac{L}{S}+\dfrac{L}{R}}. \qquad \text{XII}$$

In this formula,

$N$ = number of strokes per minute.
$L$ = length of stroke, in feet.
$S$ = cutting speed, in feet per minute.
$R$ = return speed, in feet per minute.
$\frac{R}{S}=P$, ratio of return speed to cutting speed.

The above formula can also be derived from the formula,

$$N=\frac{S}{L+\dfrac{L}{P}}.$$

*Solution*

**a** $N=\dfrac{S}{L+\dfrac{L}{P}}$.

**b** Then $N=\dfrac{1}{\dfrac{L}{S}+\dfrac{\frac{L}{P}}{S}}$, (dividing both numerator and denominator of the fraction of a by $S$)

c But $P=\frac{R}{S}$.

d And $N=\cfrac{1}{\cfrac{L}{S}+\cfrac{\frac{L}{R}}{\frac{S}{S}}}$ (substituting $\frac{R}{S}$ for $P$ in b). e Then $N=\cfrac{1}{\cfrac{L}{S}+\cfrac{\frac{LS}{R}}{S}}$

f But the fraction, $\cfrac{\frac{LS}{R}}{S}=\frac{L}{R}$, (dividing both the numerator and denominator by $S$)

g Therefore, $N=\cfrac{1}{\frac{L}{S}+\frac{L}{R}}$, substituting in e from f.

### TEST YOUR KNOWLEDGE OF THE PLANER

7 Find the cutting speed and the return speed of a planer that makes $3\frac{1}{4}$ strokes per minute, when the length of the stroke is 68 inches and the ratio is "2.4".

8 Find the number of strokes per minute for planing, when the cutting speed is 50 feet per minute, the stroke is 10 feet, and the ratio is "2".

## TIME FOR MACHINING

The *feed* of a lathe tool is the amount of side movement of the tool for each revolution of the work. If the feed is $\frac{1}{16}$ inch, for each revolution of the work, the lathe carriage with tool moves $\frac{1}{16}$ inch along the lathe bed, cutting a chip $\frac{1}{16}$ inch wide.

### *Feed of cutting tools*

The feed of a drill in the drill press is its downward movement per revolution.

The feed of a milling machine is the movement of the milling machine table for each revolution of the cutter.

Sometimes the feed is expressed as the distance which the drill or the milling machine table moves forward in one minute. In order to avoid confusion, it is always best to state plainly in each case whether feed per revolution or feed per minute is meant.

### *Time required for turning or boring work in the lathe*

To calculate the time for turning or boring in the lathe when the feed, cutting speed, and diameter of the work are known, first find the number of revolutions per minute of the work, using formula IV.

A steel bar $1\frac{1}{2}$ inches in diameter is to be turned. The length to be turned on the bar is 8 inches. The cutting speed is 40 feet per minute and the feed of the cutting tool is 0.020 inch per revolution. What is the time required to take one cut over the surface of the work?

$$N=\frac{12\times S}{D\times 3.1416}=\frac{12\times 40}{1\frac{1}{2}\times 3.1416}=\frac{480}{4.7}=102.1 \text{ (Use 102.)}$$

As the cutting tool feeds forward 0.020 inch for each revolution of the work, it is fed forward 102 × 0.020, or 2.04 inches in one minute. The time required to traverse the whole length of the work, 8 inches, is obtained by finding how many times 2.04 is contained in 8, or by dividing 8 by 2.04. The result of this division is 3.92 minutes. It would take approximately 4 minutes to traverse the work once with the speed and feed given.

This calculation, expressed in a formula, takes this form:

$$\begin{array}{l}\text{time to take one}\\ \text{cut over the work}\end{array}=\frac{\text{length of cut in inches}}{\text{rev. per min.}\times\text{feed in inches per rev.}}$$

If $T$ = time to take one cut over the work in minutes, $L$ = length of cut in inches, $N$ = revolutions of the work per minute, and $F$ = feed per revolution in inches, then the formula can be written:

$$T=\frac{L}{N\times F}.$$

Using the formula for the calculation:

$$T=\frac{8}{0.020\times 102}=\frac{8}{2.04}=3.92 \text{ minutes.}$$

In the formula above, $N\times F$ is the distance in inches the tool travels in one minute, or the feed in inches per minute.

If $F_M$ = feed in inches per minute, then the formula can be written:

$$T=\frac{L}{F_M}$$

#### TEST YOUR KNOWLEDGE OF MACHINING

**9** A cut 10 inches long is to be taken on a bar of tool steel 2 inches in diameter. How long will it take for one cut, when the cutting speed is 35 feet per minute, and a feed of $\frac{1}{32}$ inch is used?

**10** A gray iron casting 14 inches in diameter is to have a cut taken 5 inches long. Find the time to take one cut, when the feed is $\frac{1}{16}$ inch and the cutting speed is 60 feet per minute.

## *Time required for drilling*

Let us calculate the time required for drilling a given depth of hole when the cutting speed, diameter of the drill, and the feed per revolution are known.

A drill 1 inch in diameter has a cutting speed of 35 feet per minute, and a feed of 0.010 inch per revolution. Find the time to drill a hole $4\frac{1}{2}$ inches deep.

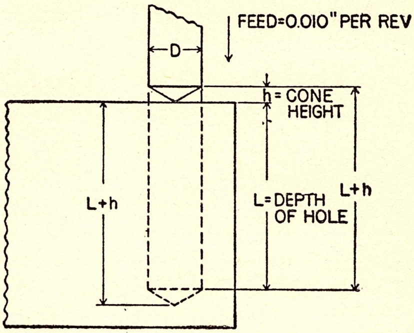



Fig. 29

From Fig. 29, we see that the total distance the drill travels downward is equal to the depth of the hole, $L$, plus the cone height, $h$, of the drill, or $L+h$. The cone height can be calculated, but it is more convenient to refer to a table. From table LIV, the cone height for a 1-inch diameter drill is 0.301 inch; thus, the distance the drill travels downward is 4.5 inches + 0.301 inch, or 4.801 inches. For calculations of this kind, 4.8 inches is sufficiently accurate.

The number of revolutions per minute can be found from formula IV:

$$N=\frac{12\times S}{D\times 3.1416}=\frac{12\times 35}{1\times 3.1416}=\frac{420}{3.1416}=133.6 \text{ rpm.}$$

The distance in inches the drill feeds downward in one minute is equal to the feed in inches per revolution times the number of revolutions per minute the drill makes; thus, $0.010\times 134=1.34$ inches. As the drill feeds downward 1.34 inches in one minute, the time required for drilling a hole 4.8 inches deep is found by dividing 4.8 by 1.34; thus, $4.8\div 1.34=3.58$ minutes. The time for drilling the hole is nearly 3.6 minutes.

If $T$ = the time required for drilling in minutes, $L$ = depth of drilled hole in inches, $N$ = number of revolutions per minute of the drill, $F$ = feed per revolution in inches, and $h$ = cone height for drill, the formula for this calculation can be written:

$$T=\frac{L+h}{F\times N}.$$

Using the above formula:

$$T=\frac{4.5+0.301}{0.010\times 134}=\frac{4.8}{1.34}=3.58 \text{ minutes.}$$

In the formula above, $F\times N=F_M$, the feed in inches per minute. The formula can also be written:

$$T=\frac{L+h}{F_M}$$

### TEST YOUR KNOWLEDGE OF DRILLING

**11** A $1\frac{1}{2}$-inch drill is used to drill a hole $2\frac{1}{2}$ inches deep. If the feed is 0.012 inches per revolution and the cutting speed is 40 feet per minute, find the time required to drill the hole.

**12** A $\frac{3}{4}$-inch diameter H.S. Steel drill is used to drill a hole in soft cast iron 6 inches deep. How long will it take to drill the hole if the feed is 0.008 inch per revolution? (*Hint*: Select cutting speed from Table LIII.)

### *Time required for milling*

The time for milling can be found if the cutting speed, the diameter of the cutter, and the feed per revolution are known.

It is required to machine a surface 8 inches long with a milling cutter 3 inches in diameter. The cutting speed is 35 feet per minute, the depth of cut is $\frac{1}{8}$ inch, and the feed is 0.052 inch per revolution.

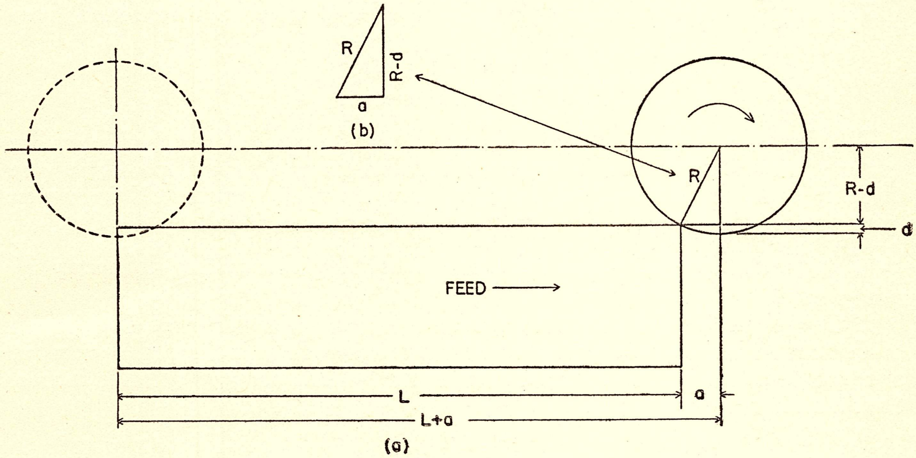



Fig. 30

From Fig. 30a, we see that the total distance the table, and also the work, travels to take one cut over the piece is equal to the length of the work, $L$, plus $a$, the approach of the cutter to the work. $L$ can be measured, but $a$ must be calculated. In Fig. 30b, the side, $a$, of the triangle is equal to the approach of the cutter to the work.

To solve the right triangle for $a$, the rule is:

$$R^2 = a^2 + (R-d)^2 \quad \text{(See page 274)}$$

Then $\quad a^2 = R^2 - (R-d)^2$

And $\quad a = \sqrt{R^2 - (R-d)^2}$.

$R$ = radius of the cutter $= 1\frac{1}{2}$ inches.

$d$ = depth of cut $= \frac{1}{8}$ inch.

$a$ = approach of cutter to work in inches.

$$a=\sqrt{\left(1\frac{1}{2}\right)^2-\left(1\frac{1}{2}-\frac{1}{8}\right)^2}$$

$$a=\sqrt{\left(1\frac{1}{2}\right)^2-\left(1\frac{3}{8}\right)^2}$$

$$a=\sqrt{(1.5\times1.5)-(1.375\times1.375)}$$

$$a=\sqrt{2.25-1.89}$$

$$a=\sqrt{0.36}$$

$$a=0.6 \text{ inches}$$

The total travel of the table is $L+a$; thus, 8 inches + 0.6 inches = 8.6 inches.

The number of revolutions per minute of the cutter can be found from formula IV:

$$N=\frac{12\times S}{D\times3.1416}=\frac{12\times35}{3\times3.1416}=\frac{420}{9.43}=44.5 \text{ rpm.}$$

The distance the table travels in one minute is equal to the feed in inches per revolution times the number of revolutions the cutter makes in one minute; thus, $0.052\times44.5=2.3$ inches. As the table travels 2.3 inches in one minute, the time required to travel 8.6 inches is found by dividing 8.6 by 2.3; thus, $8.6\div2.3=3.74$ minutes. The time for milling is nearly $3\frac{3}{4}$ minutes.

If $T$ = time for the cutter to traverse the work in minutes, $L$ = length of the cut in inches, $a$ = approach of cutter to work in inches, $N$ = number of revolutions per minute of the cutter, and $F$ = feed per revolution in inches, the formula can be written:

$$T=\frac{L+a}{F\times N}.$$

Using the formula above,

$$T=\frac{8+0.6}{0.052\times44.5}=\frac{8.6}{2.3}=3.74 \text{ minutes.}$$

In the formula above, $F\times N=F_M$, the feed in inches per minute. The formula can be written:

$$T=\frac{L+a}{F_M}.$$

#### TEST YOUR KNOWLEDGE OF MILLING

**13** A surface 2 inches long is machined with a milling cutter $2\frac{1}{2}$ inches in diameter. How long will it take, if the cutting speed is 35 feet per minute, the depth of cut is $\frac{1}{16}$ inch, and the feed is 0.046 inch per revolution?

**14** It is required to machine a surface $10\frac{1}{2}$ inches long with a milling cutter

$2\frac{1}{2}$ inches in diameter. Find the time it will take if the cutting speed is 40 feet per minute, the depth of cut $\frac{3}{16}$ inch, and the feed is 0.032 inches.

### *Time required for planing and shaping*

The feed of a planer tool is its sidewise movement for each cutting stroke of the platen. If the tool-carrying head moves along the crossrail $\frac{1}{32}$ inch for each cutting stroke, we say the feed is $\frac{1}{32}$ inch. For each cutting stroke, there is necessarily a return stroke. In the following, when the expression, *number of strokes*, is used, it means the number of cutting strokes.

The time for planing a piece of work is readily calculated if the width of the work, the number of strokes of the platen per minute, and the feed per stroke are known.

A planer makes 6 strokes per minute. The feed per stroke is $\frac{1}{8}$ inch, and the width of the work is 24 inches. Find the time required for planing the work.

As the feed per stroke is $\frac{1}{8}$ inch, and the platen makes 6 strokes per minute, the feed per minute is $\frac{1}{8}\times 6$, or $\frac{3}{4}$ inch. The total number of minutes required for the tool to traverse the work is found by dividing 24 by $\frac{3}{4}$; thus, $24\div\frac{3}{4}=24\times\frac{4}{3}=32$ minutes.

The time required for planing the work is 32 minutes. If $T$ = time required for planing in minutes, $W$ = width of the work in inches, $F$ = feed per stroke in inches, and $N$ = number of strokes per minute, then the formula can be written:

$$T=\frac{W}{F\times N}.$$

Using the above formula,

$$T=\frac{24}{\frac{1}{8}\times 6}=\frac{24}{\frac{3}{4}}=24\times\frac{4}{3}=32 \text{ minutes.}$$

In the shaper, the work table is given a sidewise movement for each cutting stroke of the tool. If the table moves along the crossrail $\frac{1}{10}$ inch for each cutting stroke of the tool, we say that the feed is $\frac{1}{10}$ inch. In the shaper as in the planer, there will be a return stroke for each cutting stroke, and the expression, *number of strokes*, means the number of cutting strokes.

If $T$ = time required for shaping in minutes, $W$ = width of the work

in inches, $F$ = the feed per stroke in inches, and $N$ = the number of strokes per minute, the formula can be written:

$$T=\frac{W}{F\times N}.$$

### TEST YOUR KNOWLEDGE OF PLANING AND SHAPING

**15** Find the time for planing a surface $4\frac{1}{2}$ inches wide, when the number of strokes per minute is 6 and the feed is $\frac{1}{16}$ inch per stroke.

**16** Find the time for planing a casting 16 inches wide, when the length of stroke is 3 feet, the feed is $\frac{5}{32}$ inch, the cutting speed is 65 per minute, and the return speed ratio is 2 to 1. (*Hint*: Find number of strokes using formula XII.)

**17** Find the time for shaping the surface of a casting 2 inches wide, when the number of strokes per minute is 60 and the feed is 0.010 inch per stroke.

## THREADS AND TAPDRILLS

The terms, *pitch* and *lead*, of screw threads are often used interchangeably, with resultant confusion. The *pitch* of a thread is the distance from the top of one thread to the top of the next thread, as shown in Fig. 31. In the National Form of Thread (Fig. 34), the pitch is defined as the distance from a point on one thread to a corresponding point on the next thread. No matter whether the screw has a single or a multiple thread, the pitch is always the distance from the top of one thread to the top of the next thread as stated above.

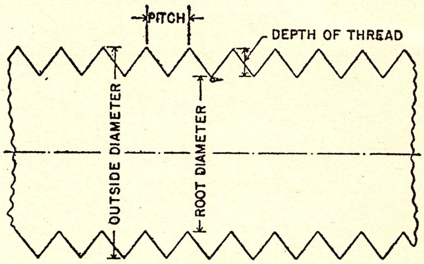



Fig. 31

The *lead* of a screw thread is the distance the screw moves forward in one complete turn, or it is the distance the nut will advance on the screw for one full revolution of the nut. In a single-threaded screw, the pitch and the lead are equal, because the nut moves forward the distance from one thread to the next thread, if turned around once. In a double-threaded screw, the nut will advance on the screw a distance equal to two threads, or twice the pitch, so that, in a double-threaded screw, the lead is twice the pitch. In a triple-threaded screw, the lead is equal to three times the pitch and, in a quadruple-threaded screw, the lead is equal to four times the pitch. The lead can also be expressed as being the distance from center to center of the same thread, after one turn. Fig. 32 shows the lead and pitch for three screws with American Acme Standard Threads. The first is a single-threaded screw, the second is double-threaded, and the third is triple-

threaded. In a single-threaded screw, the lead is the distance to the next thread from the one first considered. In a double-threaded screw, there are two threads running side by side around the screw, so that the lead is the distance to the second thread from the first one considered. In a triple-threaded screw, it is the distance to the third thread, and, in a quadruple-threaded screw, it is the distance to the fourth thread.

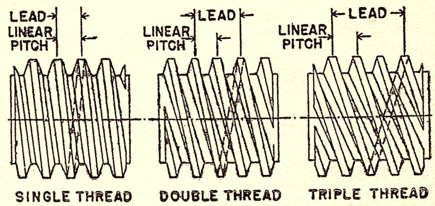



Fig. 32

The term, *pitch*, is often used improperly to denote the number of threads per inch. We hear of screws having 8-pitch thread or 10-pitch thread, when 8 threads per inch or 10 threads per inch is meant. The number of threads per inch is the number of threads counted in one inch of length, when a steel

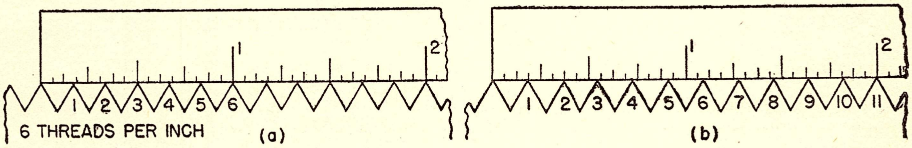



Fig. 33

scale is held against the screw, as shown in Fig. 33. Thus, at *a*, we count 6 threads per inch. Note that the thread directly under the end of the scale is not counted. If there is not a whole number of threads in one inch, count the threads in two or more inches, until the top of one thread comes opposite an inch-mark. The number of threads counted divided by the number of inches will give the number of threads per inch. In *b*, we count 11 threads in 2 inches; thus, $11 \div 2 = 5\frac{1}{2}$ threads per inch.

The pitch of a screw equals 1 divided by the number of threads per inch. Expressed in a formula:

$$\text{pitch} = \frac{1}{\text{number of threads per inch}}.$$

The number of threads per inch equals 1 divided by the pitch of the screw. Expressed in a formula:

$$\text{number of threads per inch} = \frac{1}{\text{pitch}}.$$

If $p$ = the pitch, and $n$ = the number of threads per inch, then the formula can be written:

$$p = \frac{1}{n} \text{ and } n = \frac{1}{p}.$$

If the number of threads per inch is 10, the pitch equals $\frac{1}{10}$. If the pitch equals 0.025 inches, the number of threads per inch equals $1 \div 0.025 = 40$.

In multiple-threaded screws, confusion is often caused by the indefinite designation used. One way to express that a double-threaded screw is re-

quired is to say "two threads per inch double". This means that the screw is cut with 2 separate and distinct threads. The threads are side by side, and the number of threads per inch (counting the threads with a scale as in Fig. 33) is 4. The pitch of the screw is $\frac{1}{4}$ inch. The lead for a double-threaded screw is equal to twice the pitch; thus, $2\times\frac{1}{4}=\frac{1}{2}$ inch.

To cut this screw, the lathe would be geared to cut two threads per inch, but the depth would be cut for a screw having 4 threads to the inch. "Six threads per inch triple" means that there are 6 times 3 threads per inch; thus, $6\times3=18$ threads along one inch of the screw when counted by the scale. The pitch of the screw is $\frac{1}{18}$ inch, but as the screw is triple-threaded, the lead of the thread is 3 times the pitch; thus, $3\times\frac{1}{18}=\frac{1}{6}$ inch.

Perhaps the best way to express that the above triple-threaded screw is to be cut is $\frac{1}{6}$ inch lead, $\frac{1}{18}$ inch pitch, triple-threaded.

For single-threaded screws, the number of threads per inch and the form of the thread are given.

## *American National Form Thread*

The American National Form Thread, formerly known as the "United States Standard", is one of the most widely used thread forms in the world. There are two standard series in commercial use: the N.C. (National Coarse) and the N.F. (National Fine).

The depth of the National Form Thread equals $0.6495 \times$ pitch. The width of the flat of the thread at top and bottom equals $\frac{1}{8} \times$ pitch. The minor diameter (root diameter) is found by subtracting two times the depth of the thread from the major diameter (outside diameter) of the screw.

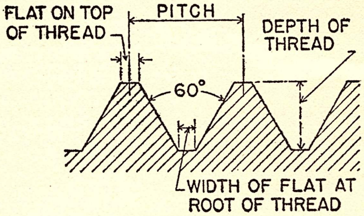



Fig. 34

## *Tap drill sizes*

The diameter of the drill used for drilling holes previous to tapping, to produce a full depth of thread, should equal the minor diameter of the thread.

Table LV (page 640) gives the depths of threads for National Form Threads. If twice the figure in the table opposite the number of threads is subtracted from the major diameter, the minor diameter is obtained.

*Illustrative Example*

Find the minor diameter for a screw $\frac{3}{4}$–10 NC.

In the table, under *National Coarse-thread Series*, opposite $\frac{3}{4}$*-inch diameter*

and 10 *threads per inch*, the depth, $h$, is 0.06495 inches. Twice this depth is $2 \times 0.06495 = 0.1299$ inches. The minor diameter equals the major diameter minus 0.1299; thus, $0.750 - 0.1299 = 0.6201$ inches. From a table of decimal equivalents (See Table IX, page 125), the nearest commercial drill size to the minor diameter is 0.625 inches, or $\frac{5}{8}$ inch. The tap-drill diameter for a $\frac{3}{4}$–10 NC tapped hole to produce a thread of nearly full depth is $\frac{5}{8}$ inch.

Lower power consumption is required and fewer tapping troubles are encountered if the tap-drill size is chosen to produce 75% of thread depth. Thread percentage times twice the depth equals $0.75 \times 2 \times 0.06495 = 0.0974$ inches. The tap drill diameter for 75% of thread depth equals $0.750 - 0.0974 = 0.6526$ inches. The nearest commercial drill size to this is 0.65625, or $\frac{21}{32}$ inch.

To figure the size of a hole prior to tapping, use the following formula:

$$\text{Tap drill size} = \text{major diameter} - \text{thread percentage} \times 2h.$$

Using the formula, we get:

$$\begin{aligned}\text{Tap drill size} &= 0.750 - (0.75 \times 2 \times 0.06495)\\ &= 0.750 - (0.75 \times 0.1299)\\ &= 0.750 - 0.0974\\ &= 0.6526 \text{ inches.}\end{aligned}$$

The nearest commercial drill size is $\frac{21}{32}$ inch.

### TEST YOUR KNOWLEDGE OF THREADS AND TAP DRILLS

**18** Find the tap drill size for a 1–8 NC tapped hole to produce a thread 75 per cent of full depth.

**19** Find the tap drill size for a $\frac{3}{4}$–16 NF tapped hole to produce a thread 65 per cent of full depth.

## CUTTING THREADS IN THE LATHE

Cutting a thread in an engine lathe is accomplished by moving the lathe carriage a definite distance along the axis of the work for each revolution of the work while the tool is cutting. If the work revolves 10 times while the carriage moves one inch along the bed of the lathe (axis of the work), 10 threads per inch will be cut on the work.

The number of times the spindle (which revolves the work) revolves while the carriage moves one inch along the lathe bed is controlled by placing gears of different sizes on the spindle stud shaft and the lead screw. Since the gears can be changed at will by the operator, they are called *change gears*.

The change gears can be arranged to form either a simple gear

train or a compound gear train, and are usually designated as simple gearing or compound gearing. (See Figs. 35 and 36.)

A simple-geared lathe has only one change of speed between the

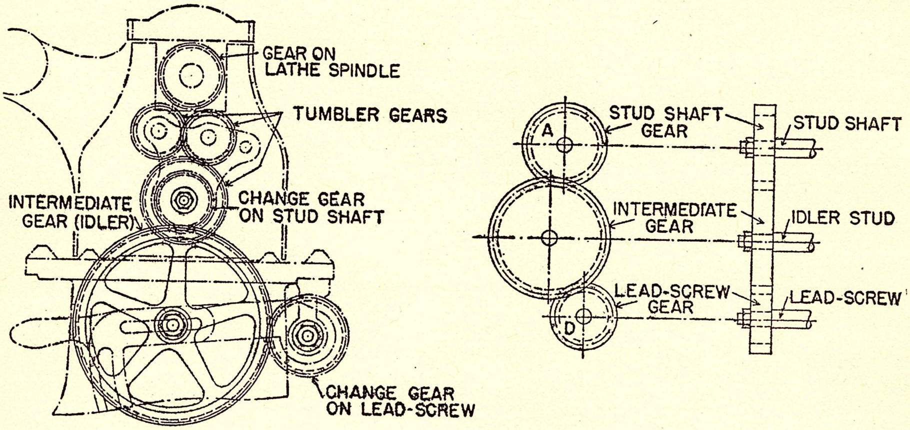



Fig. 35

stud shaft and the lead screw. When simple gearing is used, it is always necessary to use an intermediate gear between the gear on the stud shaft and the gear on the lead screw.

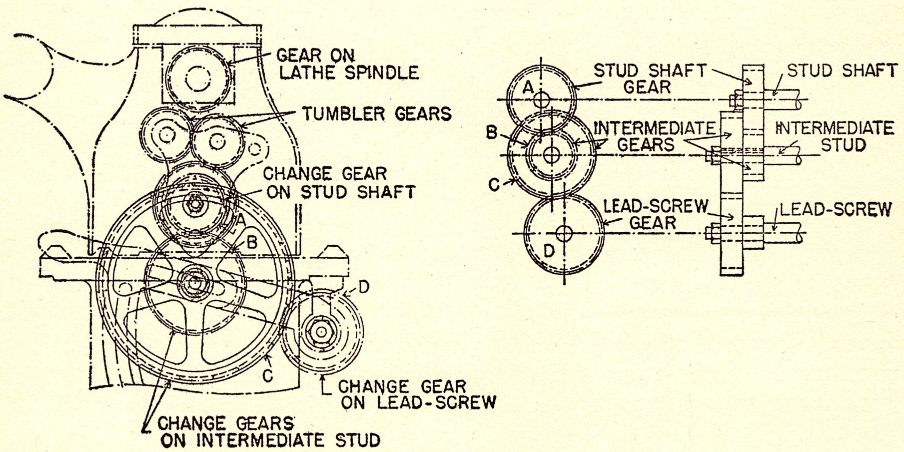



Fig. 36

The intermediate gear has no influence on the rate of turning of the lead screw and at the same time allows it to turn in the same direction as the stud shaft.

### *Finding the lead*

To make change gear calculations for the lathe, we must first find the "lead" or "lathe constant".

To find the lathe constant of a lathe, place gears with the same number of teeth on the stud shaft and on the lead screw. Then cut a thread on a piece of work in the lathe. The number of threads per inch cut on the work is called the "lathe constant" of the lathe.

*Illustrative Example*

In a certain lathe, place gears with 40 teeth on the stud shaft and on the lead screw, and any convenient gear on the intermediate stud. Then cut a thread on a piece of work placed between centers in the lathe. If the number of threads per inch when counted is found to be 6, the "lathe constant" of this lathe is said to be 6. Once the lathe constant for a given lathe is found, it is recorded. It is always the same for a given lathe.

### *Thread cutting with simple gearing*

When the "lathe constant" has been found, the number of teeth in the change gears for cutting any number of threads (within the capacity of the lathe) can be determined.

The rate of travel of the carriage compared with the revolution of the spindle is a fraction whose numerator is always the same as the "lathe constant" and whose denominator is always the same as the number of threads per inch to be cut. Multiply the numerator and denominator of this fraction by the same number (any number) to get a new fraction with a larger numerator and denominator. In this new fraction, the numerator gives the number of teeth in the gear on the stud shaft and the denominator the number of teeth in the gear on the lead screw. This rule can be expressed as a formula:

$$\frac{\text{lathe constant}}{\text{threads per inch to be cut}}=\frac{\text{teeth in gear on stud shaft}}{\text{teeth in gear on lead screw}}.$$

The gears supplied with the lathe are varied in size by adding the same number of teeth to the number of teeth in the gear next below in size. The number of teeth added is known as the *gear progression*. In many lathes, the gear progression is 4; in a few lathes, 5 and 7 are used. With a gear progression of 4, the smallest gear is usually 24 teeth, and proceeds 28, 32, 36, 40, and so on up to 100 teeth.

Eight threads per inch are to be cut in a lathe, which has a lathe constant of 6. The gear progression is 4 and the gears available are 24, 28, . . . , 100.

$$\frac{\text{lathe constant}}{\text{threads per inch to be cut}}=\frac{6}{8}=\frac{6\times 4}{8\times 4}=\frac{24}{32},\quad \frac{\text{stud gear}}{\text{lead screw gear}}.$$

By multiplying both the numerator and the denominator, we obtain two gears that are available, with 24 and 32 teeth, respectively. The 24-tooth gear is placed on the stud shaft, and the 32-tooth gear on the lead screw.

If both numerator and denominator were multiplied by 5, we should have:

$$\frac{6}{8}=\frac{6\times 5}{8\times 5}=\frac{30}{40}.$$

These two gears are not available in the change gears supplied with this lathe.

Any two gears having the same ratio can be used if available.

$$\frac{6}{8}=\frac{6\times 6}{8\times 6}=\frac{36}{48}.$$

These gears are available and could be used.

It is required to cut $11\frac{1}{2}$ threads per inch (pipe thread) in a lathe which has a lathe constant of 6. The gear progression is 4 and the gears available are 24, 28, . . . , 100.

$$\frac{6}{11.5}=\frac{6\times 4}{11.5\times 4}=\frac{24}{46}.$$

These gears are not available.

$$\frac{6}{11.5}=\frac{6\times 8}{11.5\times 8}=\frac{48}{92}.$$

These gears are available.

$$\frac{6}{11.5}=\frac{6\times 6}{11.5\times 6}=\frac{36}{69}.$$

These gears are not available on the lathe above, but, as $11\frac{1}{2}$ threads per inch is widely used, many lathes are supplied with a 69-tooth gear for this purpose.

#### TEST YOUR KNOWLEDGE OF SIMPLE GEARING

**20** A lathe has a constant of 6. If the gear progression is 4, and gears available are 24, 28, . . . , to 100, calculate the change gears for cutting 12 threads per inch.

**21** Find the change gears for cutting 20 threads per inch, when the lathe constant is 8. Gear progression and gears available are the same as in problem 20.

### *Thread cutting with compound gearing*

Because the change gears of a lathe are limited in number, it is not possible to cut all numbers of threads per inch by simple gearing, and compound gearing must be used.

The same method is used in compound gearing as for simple gearing, except that both the numerator and the denominator of the fraction are divided into two factors, one factor in the numerator and one in the denominator making one pair.

*Illustrative Problem*

Using the same lathe constant and gear progression as in simple gearing, cut 32 threads per inch.

Our fraction is $\frac{6}{32}$.

Dividing the numerator and the denominator of the fraction into two factors and multiplying the numerator and the denominator of each pair by the same number as shown below we get,

$$\frac{6}{32}=\frac{2\times3}{4\times8}=\frac{(2\times16)\ (3\times12)}{(4\times16)\ (8\times12)}=\frac{32\times36}{64\times96}.$$

The four numbers in the last fraction give the numbers of teeth in gears to be used. These gears are available and are placed as follows: the gears in the numerator, with 32 and 36 teeth, are the driving gears, and those in the denominator, with 64 and 96 teeth, are driven gears. In Fig. 36, the driving gears are gear *A* on the stud shaft, and gear *C* the second gear on the intermediate stud, which meshes with the lead screw gear. Driven gears are gear *B* on the intermediate stud, which meshes with the stud shaft gear, and the lead screw gear, *D*, which meshes with the second gear, *C*, on the intermediate stud.

The formula for calculating compound change gears can be written as follows:

$$\frac{\text{lathe constant}}{\text{threads per inch to be cut}}=\frac{\text{product of teeth in driving gears}}{\text{product of teeth in driven gears}}.$$

Using the compound gearing, find the change gears to cut 36 threads per inch when using same lathe as above.

$$\frac{6}{36}=\frac{2\times3}{4\times9}=\frac{(2\times16)\ (3\times8)}{(4\times16)\ (9\times8)}=\frac{32\times24}{64\times72}.$$

The gears with 32 and 24 teeth are the driving gears and the 64- and 72-tooth gears are driven gears.

From Fig. 37, we see that the gears with 32 and 64 teeth are mounted on the same shaft and revolve together—that is, when one makes a complete turn, the other also makes a complete turn. The compound gear (as it is called) is made up of two gears, one having twice as many teeth as the other. The effect of the compound gear is to change the speed ratio 2 to 1. With the compound gear in the gear train, the lead screw will turn only one-half as fast as it did before and the number of threads cut will be twice as many as when the same gears are used on the stud shaft and lead screw in simple gearing; thus, to cut 36 threads per inch compound gearing, divide the number of threads per inch to be cut by 2 and use the formula for simple gearing.

$$36\div2=18.$$

$$\frac{6}{18}=\frac{6\times4}{18\times4}=\frac{24}{72},\ \frac{\text{stud gear}}{\text{screw gear}}.$$

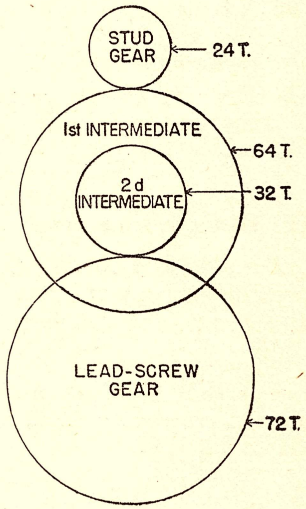



Fig. 37

The compound gear can be any two gears in which the large gear has twice as many teeth as the small one; thus, the compound gear could be 32 and 64, 24 and 48, 28 and 56, etc.

### TEST YOUR KNOWLEDGE OF COMPOUND GEARING

**22** It is required to cut 30 threads per inch on an engine lathe with a constant of 8 and a gear progression of 4. The change gears available are 24, 28, . . . , 80. Find the change gears using compound gearing.

**23** Find the change gears for cutting 40 threads per inch on the lathe in problem 22. Use compound gearing.

## TAPER CALCULATIONS

*Taper* is the difference in the diameter of a cylindrical piece of work. Taper is usually expressed as taper per inch or taper per foot.

Taper per inch is the difference between the two diameters in a tapering piece one inch long.

In Fig. 38, the diameter at the small end is $\frac{1}{2}$ inch, and the diameter at the large end $\frac{9}{16}$ inch, and the length of the piece is 1 inch; the taper, therefore, is $\frac{1}{16}$ inch per inch.

Taper per foot is the difference between the two diameters in a tapering piece 1 foot long.

In Fig. 39, the diameter at the small end is $1\frac{1}{4}$ inches, and the diameter

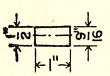



Fig. 38

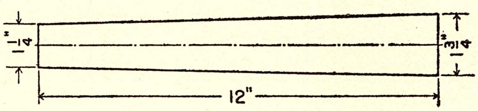



Fig. 39

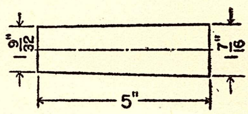



Fig. 40

at the large end $1\frac{3}{4}$ inches, and the length of the piece is one foot. The taper is $\frac{1}{2}$ inch in one foot of length, or $\frac{1}{2}$ inch per foot.

In Fig. 40, the diameter at the small end is $1\frac{9}{32}$ inches, and at the large end $1\frac{7}{16}$ inches, and the length is 5 inches. The difference in the diameters is $1\frac{7}{16}-1\frac{9}{32}$, or $\frac{5}{32}$ inch, and the taper is $\frac{5}{32}$ inch in 5 inches. If the taper in a given length is known, the taper per inch can be found. If the taper is $\frac{5}{32}$ inch, and the length is 5 inches, the taper per inch equals the taper in 5 inches divided by 5, or (in this case) $\frac{5}{32}\div 5=\frac{1}{32}$, which is the taper per inch. The taper per foot is found by multiplying the taper per inch by 12. The taper per foot in this case is $12\times\frac{1}{32}=\frac{3}{8}$ inch.

The length *is always measured along the center line* (*axis*) *of the work or parallel to it*, and never along the tapered surface.

What has been stated above may be shown more clearly in Fig. 41. The

length of the tapered piece is 1 foot, and the diameters at the small and large ends are 1.000 inch, and 1.600 inch, respectively. The difference in the diameters in one foot of length equals 0.600 inch, and the taper per foot is said to be 0.600 inch per foot. The taper per inch is found by dividing

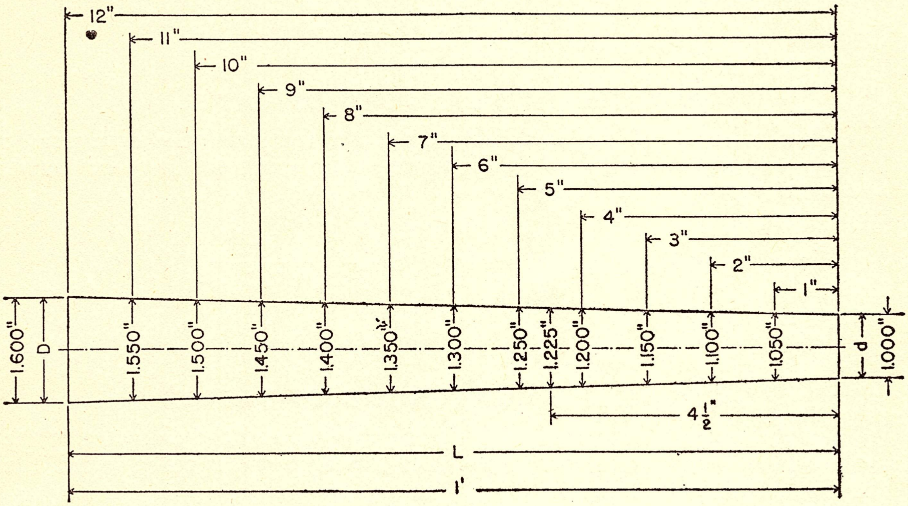



Fig. 41

the taper per foot by 12; thus, $0.600 \div 12 = 0.050$ inch. The taper per inch is said to be 0.050 inch per inch.

From Fig. 41, we see that, at a distance 1 inch from the small end, the diameter would be increased by an amount equal to the taper per inch or $1.000 + 0.050 = 1.050$ inches. At two inches from the small end, the diameter would be increased by $2 \times 0.050 = 0.100$ inches. At two inches from the small end, the diameter would be $1.000 + 0.100 = 1.100$ inches. Thus, we see that the diameter increases 0.050 inch for each inch of length, and in one foot, or 12 inches, the diameter increases $12 \times 0.050$ inch, or 0.600 inch. Therefore, the amount of taper for any given length is equal to the taper per inch times the length of the taper in inches. From Fig. 41, we see that the taper in $4\frac{1}{2}$ inches is $4.5 \times 0.050$, or 0.225 inch, and the diameter $4\frac{1}{2}$ inches from the small end $= 1.000 + 0.225 = 1.225$ inches.

## *Formulas for calculating taper*

The formulas for taper per inch and taper per foot can be written:

$$\text{taper per inch} = \frac{\text{large diameter} - \text{small diameter}}{\text{length of work in inches}}$$

$$\text{taper per foot} = \frac{\text{large diameter} - \text{small diameter}}{\text{length of work in inches}} \times 12$$

If $T_{ft.}$ = taper per foot, $T_{in.}$ = taper per inch, $L$ = length of work in

inches, $D$ = diameter at large end in inches, and $d$ = diameter at small end in inches, then the above formulas can be written:

$$T_{in.} = \frac{D-d}{L}$$

$$T_{ft.} = \frac{D-d}{L} \times 12$$

## DIAMETER AT SMALL END OF TAPER

When the length of the work, the taper per foot, and the diameter at the large end are known, and the diameter at the small end is to be found, the formula below can be used:

$$\text{diameter small end} = \text{diameter large end} - \left(\frac{\text{taper per foot}}{12} \times \text{length}\right)$$

or $d = D - \left(\frac{T_{ft.}}{12} \times L\right)$

## DIAMETER AT LARGE END OF TAPER

When the length of the work, the taper per foot, and the diameter at the small end are known, and the diameter at the large end is to be found, the following formula can be used:

$$\text{diameter large end} = \text{diameter small end} + \left(\frac{\text{taper per foot}}{12} \times \text{length}\right)$$

or $D = d + \left(\frac{T_{ft.}}{12} \times L\right)$

## DISTANCE BETWEEN TWO DIAMETERS ON A TAPERED PIECE

If the taper per foot is given, and the diameter at the small and the large ends are known, the length of the piece can be found from the formula below:

$$\text{length} = \frac{\text{diameter large end} - \text{diameter small end}}{\text{taper per foot} \div 12}$$

or $L = \frac{D-d}{T_{ft.} \div 12}$

*Illustrative Example*

A cylindrical piece of work measures $\frac{7}{8}$ inch diameter at the small end and $1\frac{1}{8}$ inches diameter at the large end. The taper is 0.6 inch per foot. Find the length of the taper.

The formula for the length between two diameters is:

$$L = \frac{D-d}{T_{ft.} \div 12}$$

Substituting the given values in this formula,

$$L=\frac{1\frac{1}{8}-\frac{7}{8}}{0.6\div 12}=\frac{0.250}{0.050}=5 \text{ inches}$$

In the denominator of the above formula, taper per foot÷12, equals taper per inch. This formula can also be written:

$$L=\frac{D-d}{T_{in.}}$$

**TEST YOUR KNOWLEDGE OF TAPER**

24 The taper on a No. 4 Morse Taper Reamer is 0.623 inch per foot. What is the length of the tapered part, if the diameter at the small end is 1.020 inches and the diameter at the large end is 1.293 inches?

25 The diameter of a tapered piece is 0.778 inch at the small end and 0.991 inch at the large end. What is the taper per foot, if the length of the tapered part is 4.246 inches?

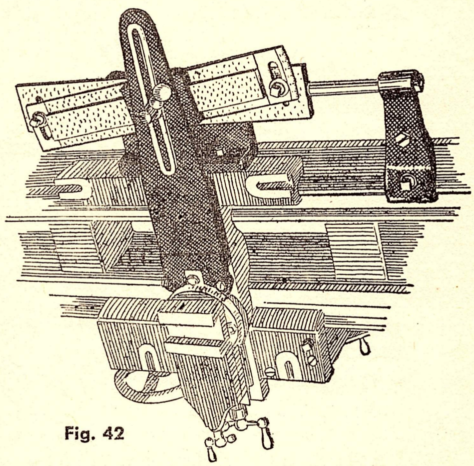


Fig. 42

### *Setting the taper attachment*

The taper attachment (See Fig. 42) is an accessory developed especially for turning tapers. The taper attachment permits a wide range of tapers to be cut, and has largely displaced the offset tailstock method (setover method) of turning tapers. In this method, the lathe centers are always in alignment, and the taper attachment, once set, reproduces the same taper no matter what the length of the work. It can

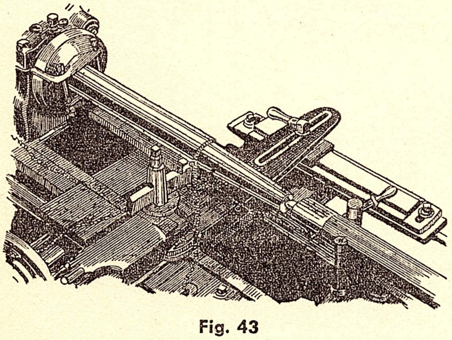


Fig. 43

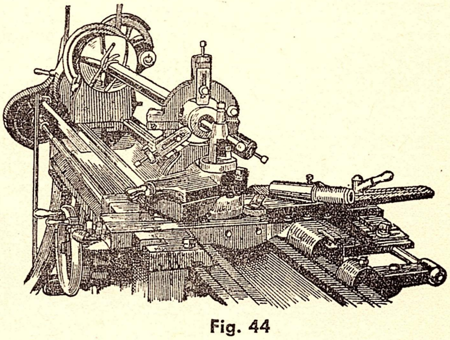


Fig. 44

be used to produce both inside and outside tapers. (See Figs. 43 and 44.) The swivel bar, which controls the taper, is graduated at one end in inches per foot of taper (taper per foot). (See Fig. 45.)

To set the taper attachment, we must know the taper per foot so that we may set the swivel bar.

To produce the taper in Fig. 46, by using a taper attachment, we must find the taper per foot.

$$T_{ft.}=\frac{D-d}{L}\times 12$$

$$T_{ft.}=\frac{1\frac{1}{8}-\frac{7}{8}}{5}\times 12$$

$$T_{ft.}=\frac{0.250}{5}\times 12$$

$$T_{ft.}=0.050\times 12$$

$$T_{ft.}=0.600$$

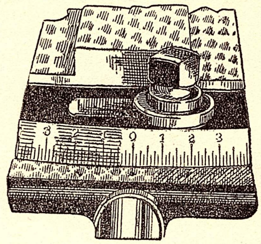


Fig. 45

From a table of decimal equivalents, we find that 0.600 inch $=\frac{39}{64}$ inch, almost.

To set the taper attachment to cut the above taper, move the swivel slide until the zero mark on the stationary base coincides with $\frac{39}{64}$ inch on the swivel slide. Take trial cut and make any slight adjustment of swivel slide necessary to produce taper to the accuracy desired.

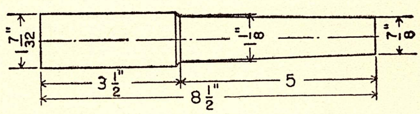



Fig. 46

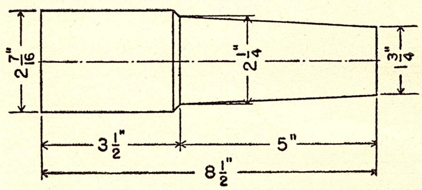



Fig. 47

**TEST YOUR KNOWLEDGE OF TAPER ATTACHMENTS**

**26** A piston rod is $38\frac{1}{2}$ inches long, and tapered at one end for 6 inches. If the diameter at the large end of the taper is $3\frac{3}{4}$ inches, and the diameter at the small end is $3\frac{1}{2}$ inches, find the setting for the taper attachment.

**27** Find the setting for the taper attachment to cut the taper in Fig. 47.

## *Setting the tailstock for turning tapers*

Fig. 48 shows the tailstock of the lathe set over for turning a taper. The centers of a lathe are in perfect alignment when the cutting tool, mounted in the tool post of the lathe carriage, travels parallel to the center line of the lathe. If a piece of work is then placed between centers and revolved, and a cut taken over it, a cylindrical piece is generated. The cylinder will have the same diameter throughout its entire length, or, we say, it is turned "straight".

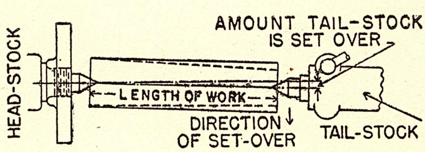



Fig. 48

If the position of the tailstock center be changed by moving the tail center out of alignment with the live center any amount, $a$, as shown in Fig. 49, the center of the work at the tail center end will be nearer to the line of travel, $cd$, of the tool than the center of the work at the live center end, and a tapered piece is generated. Setting over the tailstock is a common method for turning tapered work, especially when the lathe is not equipped with a taper attachment.

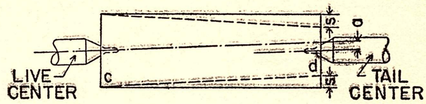


Fig. 49

The amount of taper depends on the entire length of the work between centers, and the set-over of the dead center in each case. When the tail center is set over the amount, $a$, as in Fig. 49, the radius at the small end will be smaller than the radius at the large end by the amount, $s$. The dimension, $s$, is equal to $a$, the amount the tail center has been set over, and the taper of the work in the length between centers is twice the amount the tail center is set over, or the set-over is one-half of the taper in the length of the work between centers. The length of the work is always measured along the center line of the work or parallel to it, and never along the tapered surface. (See Fig. 48.)

## FINDING THE SET-OVER

When the work is tapered its entire length, the set-over can be found if the taper per foot and the length are known.

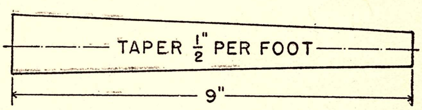


Fig. 50

In Fig. 50, the taper per foot is $\frac{1}{2}$ inch, the length is 9 inches, and the set-over is to be found. First find how much the work tapers in 9 inches. This can be found by dividing $\frac{1}{2}$ inch by 12, and multiplying the result by 9.

$$\frac{1}{2} \div 12 \times 9 = \frac{3}{8}$$

The taper in 9 inches is $\frac{3}{8}$ inch and the tail center is set over one-half of this amount. The set-over is $\frac{3}{16}$ inch.

Thus, the formula for finding the set-over, when the taper per foot and the length of the work are known, is:

$$\text{set-over} = \frac{1}{2}\left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right)$$

If $a$ = amount of set-over, $T_{ft.}$ = the taper per foot, and $L$ = length of the work, the formula can be written:

$$a = \frac{1}{2}\left(\frac{T_{ft.}}{12} \times L\right)$$

It is not practicable to calculate the set-over of the tail center so accurately that the taper can be produced to exact dimensions without taking a trial cut. This is because the lathe centers enter the work to support it. In our

calculations, we have made no allowance for this, but have assumed that the distance between the points of the centers and the length of the work are the same. The calculation for the set-over gives a close approximation; after a trial cut, we can make the necessary adjustments of the tail center to produce the correct taper.

When the work tapers its entire length and the diameters at both the large and the small ends of the work are known, the set-over can be found without knowing the taper per foot. All that it is necessary to know is the taper in the length between the centers in the lathe.

In Fig. 51, the diameters at the large and the small end are $\frac{7}{8}$ inch and $\frac{1}{2}$ inch respectively, and the length is 9 inches.

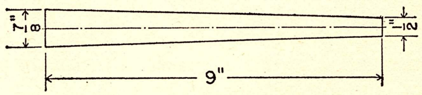



Fig. 51

The taper is the difference between the diameter at the large end and the small end. Thus, the taper in 9 inches of length is $\frac{7}{8}-\frac{1}{2}$, or $\frac{3}{8}$ inch. The set-over is one-half the taper. The set-over is $\frac{3}{16}$ inch, and the formula for the above calculation can be written:

$$\text{set-over}=\frac{1}{2}\ (\text{large diameter}-\text{small diameter})$$

$$\text{or} \qquad a=\frac{1}{2}\ (D-d).$$

When part of the work is turned straight and part of it turned tapered, the taper in the entire length of the work must be found. The set-over is one-half of this amount.

In Fig. 52, the diameter at the small end of the taper is $\frac{1}{2}$ inch. It is tapered for 6 inches, and the diameter at the large end of the taper is $\frac{3}{4}$ inch. It is then turned straight for the remaining 3 inches to a diameter of $\frac{7}{8}$ inch, the total length being 9 inches.

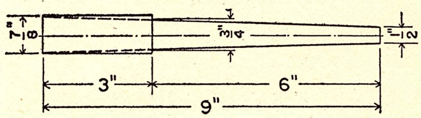



Fig. 52

First find what the taper would be in 9 inches when cut to the same taper as is required in 6 inches. The set-over is one-half of this amount. The taper in 6 inches is $\frac{3}{4}-\frac{1}{2}=\frac{1}{4}$ inch. The taper per inch equals $\frac{1}{4}\div 6$, or $\frac{1}{24}$ inch, and the taper in 9 inches equals $9\times\frac{1}{24}$, or $\frac{3}{8}$ inch. The set-over is one-half of this amount; thus, $\frac{3}{8}\div 2=\frac{3}{16}$ inch. If the taper in Fig. 52 be continued beyond the 6-inch length, as shown by the broken line, Fig. 52 would become the same as Figs. 50 and 51. The

following formulas can be used when part of the work is straight and part tapered:

$$\text{set-over} = \frac{1}{2} \times (\text{taper per inch} \times \text{total length})$$

$$\text{or,} \quad a = \frac{1}{2} \times (T_{in.} \times L)$$

$$\text{set-over} = \frac{1}{2} \times \left( \frac{\text{taper per foot}}{12} \times \text{total length} \right)$$

$$\text{or,} \quad a = \frac{1}{2} \times \left( \frac{T_{ft.}}{12} \times L \right)$$

It is important to note that, in the set-over method, the entire length of the work must be used in calculating the taper. The set-over is one-half of this amount.

#### TEST YOUR ABILITY TO FIND THE SET-OVER

**28** A piece of work $9\frac{1}{2}$ inches long is tapered for 6 inches from one end. The diameters at the large and the small end of the taper are $1\frac{5}{8}$ and $1\frac{1}{4}$ inches, respectively. Find the set-over for the tailstock.

**29** A piece of work 24 inches long is tapered at one end. The diameters at the large and the small end of the taper are 2 and $1\frac{1}{2}$ inches, respectively. If the taper per foot is $1\frac{1}{2}$ inches, what set-over is required?

### THE INDEX HEAD

The mechanism known as the *index head* is mounted on the milling machine table and is used to perform a wide variety of jobs. In cutting the teeth in a gear, we must have each tooth properly spaced in relation to the other teeth in the gear. This process of spacing the teeth in a gear, or of dividing

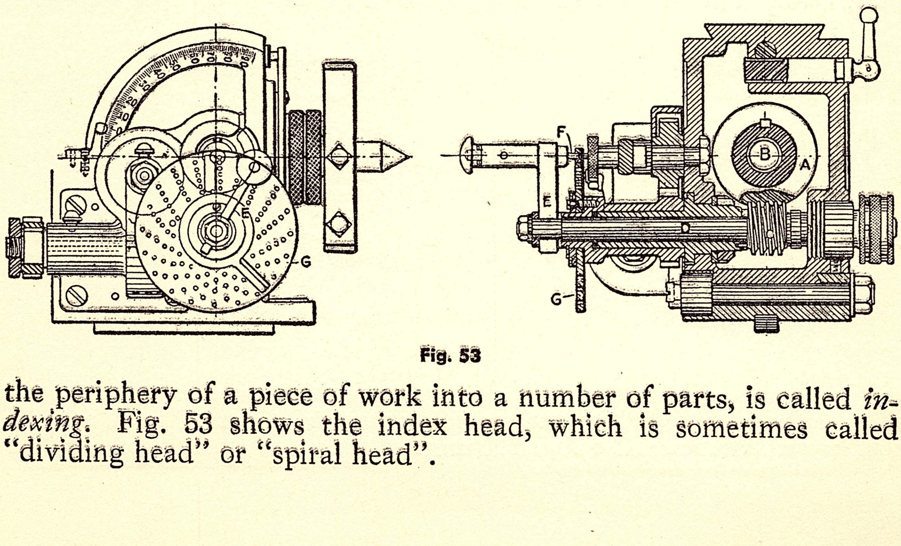



Fig. 53

the periphery of a piece of work into a number of parts, is called *indexing*. Fig. 53 shows the index head, which is sometimes called "dividing head" or "spiral head".

A worm gear (worm-wheel), *A*, is mounted securely on the index head spindle, *B*. The worm, *C*, which is an integral part of shaft *D* meshes with the worm gear, *A*. Secured to shaft *D*, at the opposite end from the worm, *C*, is the crank, *E*, in the outer end of which is fitted a pin, *F*. The end of the pin has a cylindrical projection which fits into the holes of the index plate, *G*. When the crank, *E*, is moved, the worm, *C*, is rotated, and imparts motion to the worm wheel, *A*, and the work held in the index head is rotated.

By moving the crank with the index pin a definite number of holes in one of the index circles, we impart a certain movement to the spindle and the work. Calculations for indexing consist of finding how much the crank, *E*, is required to be turned to produce the required movement for indexing the work.

Fig. 54 shows a schematic diagram of the index plate and mechanism.

## *Calculations for indexing*

Most index heads are constructed with a single thread worm engaging a worm gear having 40 teeth. When the index crank, *E*, makes one full turn, the worm, *C*, is also revolved one full turn, and this moves the worm gear one tooth, or $\frac{1}{40}$ of its circumference. In order to turn the worm gear and the spindle on which it is mounted one whole revolution, we have to turn the index crank 40 revolutions; thus, the ratio between the index crank and the spindle is 40 to 1. If we are required to revolve the spindle (work) one-half revolution, we should have to turn the index crank $\frac{1}{2}\times 40$, or 20 revolutions. If we are required to revolve the spindle $\frac{1}{4}$ revolution, we turn the index crank $\frac{1}{4}\times 40$, or 10 revolutions.

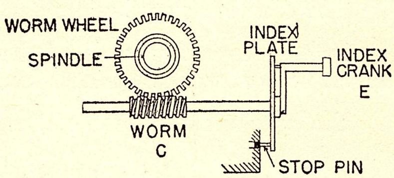


Fig. 54

Fig. 55 shows a circular piece of work with 10 equally-spaced notches cut about its periphery. To revolve the spindle (work) once requires 40 revolutions of the index crank. To revolve the spindle $\frac{1}{10}$ of a revolution would require as many turns of the index crank as $40 \div 10$, or 4. After the first notch is cut, the index crank would be turned 4 revolutions, thus causing the spindle (work) to be revolved $\frac{1}{10}$ of a revolution. The second notch is then cut and the process repeated until all of the notches have been cut. In a like manner, a piece of work with 20 notches would require $40 \div 20$, or 2, turns of the index crank to revolve the

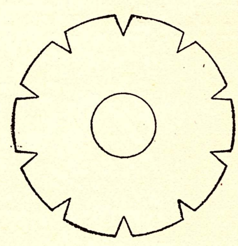


Fig. 55

work $\frac{1}{20}$ revolution; if 80 notches were required, the movement of the index crank would be $\frac{40}{80}$, or $\frac{1}{2}$ turn, to revolve the work $\frac{1}{80}$ revolution.

Thus, we see that 40 is the *index head constant*, and 40 divided by the number of divisions required equals the revolutions of the index crank to move the spindle (work) the required amount.

| Constant | | Divisions Required | | Turns of Index Crank for Each Division |
|---|---|---|---|---|
| 40 | ÷ | 10 | = | 4 |
| 40 | ÷ | 20 | = | 2 |
| 40 | ÷ | 40 | = | 1 |
| 40 | ÷ | 80 | = | $\frac{1}{2}$ |

## HOW TO USE THE INDEX PLATE AND SECTOR

In indexing, we are often required to make only a part of a turn of the index crank to get the required movement of the spindle.

It is required to cut a gear having 65 teeth. Find the proper index circle and the setting for the sector.

$$40 \div 65 = \frac{40}{65}.$$

In this case, the division is indicated by the fraction, $\frac{40}{65}$, meaning that $\frac{40}{65}$ of a whole turn of the index crank is required to revolve the gear $\frac{1}{65}$ of a revolution. This fractional turn of the index crank is accomplished by making use of the index plates supplied with the index head.

Most index heads are supplied with 3 index plates, each having 6 index circles. The plates and circles given below are those regularly supplied with the Brown & Sharpe Manufacturing Company's index head.

| Plate | Circles |
|---|---|
| 1 | 15-16-17-18-19-20 |
| 2 | 21-23-27-29-31-33 |
| 3 | 37-39-41-43-47-49 |

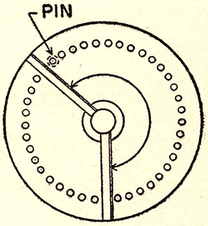



Fig. 56

From the circles available, we find that 65 is missing. It is possible to change the fraction, $\frac{40}{65}$, to a new fraction having a denominator that is the same as one of the index circles available, however. Thus,

$$\frac{40 \div 5}{65 \div 5} = \frac{8}{13} \text{ and } \frac{8 \times 3}{13 \times 3} = \frac{24}{39}.$$

We find, from the table of index circles, that 39 is available. Plate 3 would be mounted on the index head and the index crank moved 24 holes in the 39-hole circle. Fig. 56 shows the arms of the sector set for 24 holes in the

39-hole circle. (*Caution*: Never count the hole in which the pin of the index crank is placed. For 24 holes, there should actually be 25 holes between the arms of the sector, but one is filled by the pin.)

In the fraction representing the part of a turn of the index crank, the denominator is the number of holes in the circle to be used and the numerator is the number of holes the index crank is moved.

| Constant | | Divisions Required | | Crank Turns | Whole Turns | No. of Holes | Circle |
|---|---|---|---|---|---|---|---|
| 40 | ÷ | 6 | = | $6\frac{4}{6}$ | 6 | 12 | 18 |
| 40 | ÷ | 65 | = | $\frac{40}{65}$ | 0 | 24 | 39 |
| 40 | ÷ | 8 | = | 5 | 5 | 0 | Any |
| 40 | ÷ | 9 | = | $4\frac{4}{9}$ | 4 | 12 | 27 |
| 40 | ÷ | 5 | = | 8 | 8 | 0 | Any |
| 40 | ÷ | 85 | = | $\frac{8}{17}$ | 0 | 8 | 17 |

#### TEST YOUR ABILITY TO FIND THE INDEX CRANK

**30** It is required to cut 9 flutes regularly spaced in a reamer. Find the movement for the index crank.

**31** It is required to cut a gear having 55 teeth. Find the movement for the index crank.

### CALCULATIONS FOR INDEXING FOR ANGLES

In Fig. 57, a circular piece of work is shown with two cuts, the angle between the cuts being $34\frac{1}{2}$ degrees. Here the work is not required to make a complete revolution, as is done when a definite number of regularly-spaced notches are required, as in Fig. 55. Instead, we are required to find the movement of the index crank necessary to produce the required movement of the work in degrees before another cut is taken. Indexing for angles is required whenever the angle given is not a simple fraction of the whole circle (as, for example, 90 degrees, which is $\frac{1}{4}$ of a whole turn; or 72 degrees, which is $\frac{1}{5}$ of a whole turn; or 36 degrees, which is $\frac{1}{10}$ of a whole turn). The number of turns of the index crank in these cases is determined as explained before but, if the angle required is $34\frac{1}{2}$ degrees, the calculations for the indexing movement must be carried out as follows:

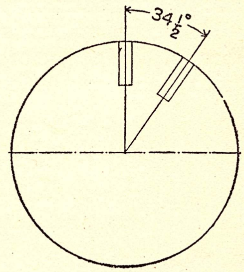


Fig. 57

Since there are 360 degrees in one complete circle, when the spindle (work) makes one revolution, the spindle revolves through 360 degrees; thus, if it requires 40 turns of the crank, to produce one revolution of the spindle (360

degrees), one turn of the index crank must move the spindle through an angle equal to 360 degrees divided by 40, or 9 degrees. Of the index circles available, 18 and 27 are divisible by 9; thus, $18 \div 9 = 2$ and $27 \div 9 = 3$ gives the number of holes to move the crank in the respective circles, to move the spindle 1 degree.

If the index crank is moved 2 holes in the 18-hole circle to index the spindle 1 degree, then 1 hole in the 18-hole circle indexes the spindle $\frac{1}{2}$ degree. In a like manner, 3 holes in the 27-hole circle indexes the spindle 1 degree; 1 hole, $\frac{1}{3}$ degree; and 2 holes, $\frac{2}{3}$ degree; thus, if the angle contains a half degree, the plate with the 18-hole circle is used. To index $34\frac{1}{2}$ degrees, proceed as follows:

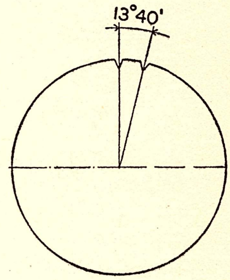


Fig. 58

$34\frac{1}{2}$ degrees = required angle

$-27$ degrees = 3 whole turns of crank

$7\frac{1}{2}$ degrees = fractional turn of crank required

$7\frac{1}{2}$ degrees = 7 degrees + $\frac{1}{2}$ degree = 14 holes + 1 hole

$34\frac{1}{2}$ degrees = 3 turns + 15 holes in the 18-hole circle

In Fig. 58, the required angle is 13°40′. As 40′ is $\frac{2}{3}$ degree, the plate with the 27-hole circle would be used.

$13\frac{2}{3}$ degrees = required angle

$-\ 9$ degrees = 1 whole turn of crank

$4\frac{2}{3}$ degrees = fractional turn of crank

$4\frac{2}{3}$ degrees = 4 degrees + $\frac{2}{3}$ degree = 12 holes + 2 holes

13°40′ = 1 turn + 14 holes in the 27-hole circle.

### TEST YOUR KNOWLEDGE OF THE INDEX HEAD

**32** It is required to have an indexing movement of $10\frac{1}{2}$ degrees. Find the plate, circle, and number of turns of the crank.

**33** Find the plate, circle, and number of turns of the crank to have an indexing movement of 38°40′.

## DIFFERENTIAL INDEXING

Only a limited number of divisions can be indexed by using the three plates regularly supplied with the index head for plain indexing. Many divisions not obtainable by plain indexing can be indexed by the differential indexing process. In this process, it is possible to have the plate and the crank revolve at the same time. The movement of the crank and plate is obtained by a train of gears interposed between the spindle of the index head and the worm shaft which imparts motion

to the plate. By this arrangement, the index crank is moved in the same circle of holes and the operation is like plain indexing.

A simple explanation will serve to show the problem involved in making the computation.

If gears are placed on the spindle, $S$, and the worm shaft (usually called work), $W$, of the index head (Fig. 59) and one intermediate gear used to complete the train, the plate will rotate in the same direction as the crank. With two intermediate gears, the plate will rotate in a direction opposite to the crank.

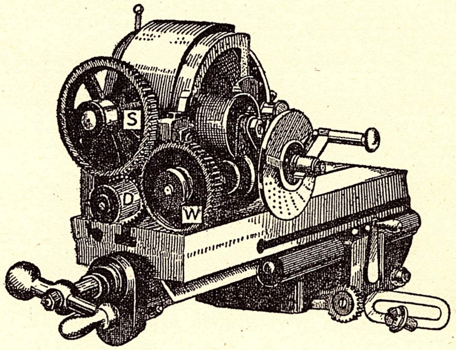


Fig. 59

If the index plate is stationary, the index crank will pass the point, $P$, 40 times (Fig. 60) for one complete turn of the spindle. With gears having the same numbers of teeth placed on the spindle, $S$, and the worm, $W$ (Fig. 59), and one intermediate gear, the plate will rotate in the same direction as the crank. In this case, the crank will pass the point, $P$, only 39 times for one turn of the spindle. This is because the plate, in making one complete turn, makes up for the fortieth turn, or one division is subtracted from the regular indexing.

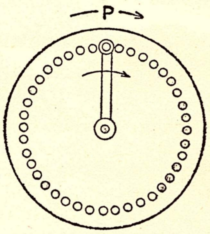


Fig. 60

In like manner, if two intermediate gears are used to complete the train, the crank will rotate in the opposite direction to the plate and the crank will pass the point, $P$, 41 times for one turn of the spindle, in order to make up for the one revolution of the plate in opposite direction, and one division is added to the regular indexing.

Thus, it is shown that the total movement of the crank at every indexing is equal to its movement relative to the plate, plus the movement of the plate when the plate rotates in the same direction as the crank, and minus the movement of the plate when the plate rotates in a direction opposite to the crank. By using the proper change gears, we can obtain the desired movement of the plate for each indexing.

In practice, the matter is even simpler than it appears in the explanation. By following through the steps discussed in the accompanying illustrative problem, the reader will have little difficulty in determining the procedure.

*Illustrative Problem*

It is required to find the indexing to cut a gear with 250 teeth.

**a** Select the index circle. (As the regularly supplied index plates are to be used, select a number of divisions that can be obtained by plain indexing and find the required movement of the crank. The number of divisions

selected can be either larger or smaller than the divisions to be indexed.) Thus,

$$250 = \text{number of divisions required}$$
$$240 = \text{number of divisions selected to find index circle}$$

$$40 \div 240 = \frac{40}{240} = \frac{1}{6} \text{ and } \frac{1\times 3}{6\times 3} = \frac{3}{18}$$

Use plate #1 and set the sector arms for a crank movement of 3 holes in the 18-circle.

**b** Find the change gears to produce the required movement of the plate. The required movement of the spindle (work) is $\frac{1}{250}$ revolution for each cut. The required movement of the crank to rotate the spindle $\frac{1}{250}$ revolution is $40\times\frac{1}{250}$, or $\frac{40}{250}$ revolution.

The movement of the plate is the difference between the crank movement for indexing the number of divisions selected (240) and the number of divisions required (250).

$$\frac{40}{240} - \frac{40}{250} = \frac{40\div 40}{240\div 40} - \frac{40\div 10}{250\div 10} = \frac{1}{6} - \frac{4}{25}$$

The common denominator of these two fractions is $6\times 25$. Subtracting, we get:

$$\frac{1}{6} - \frac{4}{25} = \frac{25}{6\times 25} - \frac{24}{6\times 25} = \frac{1}{6\times 25}$$

The required movement of the plate is $\frac{1}{6\times 25}$ revolution, while the spindle makes $\frac{1}{250}$ revolution.

As motion is imparted to the plate from the spindle, the change gears necessary to revolve the plate $\frac{1}{6\times 25}$ revolution while the spindle revolves $\frac{1}{250}$ revolution can be found.

Multiply both fractions by the same quantity to reduce them to smaller fractions, and repeat the operation if necessary to change them to whole numbers.

$$\frac{1}{6\times 25}\times(6\times 25) = \frac{6\times 25}{6\times 25} = 1 \text{ revolution of plate.}$$

$$\frac{1}{250}\times(6\times 25) = \frac{6\times 25}{250} = \frac{150}{250} = \frac{3}{5} \text{ revolution of spindle.}$$

And

$$1\times 5 = 5 \text{ revolutions of plate.}$$

$$\frac{3}{5}\times 5 = 3 \text{ revolutions of spindle.}$$

As the number of teeth in the gears is inversely proportional to the number of revolutions the gears make, the teeth in the gears are found by inverting the number of revolutions they make. Thus,

$$\frac{5 \text{ revolutions of plate}}{3 \text{ revolutions of spindle}} = \frac{3 \text{ teeth in worm gear}}{5 \text{ teeth in spindle gear}}$$

The change gears supplied have numbers of teeth as follows: 24 (2), 28, 32, 40, 44, 48, 56, 64, 72, 86, 100.

$$\frac{3}{5}=\frac{3\times 8}{5\times 8}=\frac{24}{40},\ \frac{\text{worm gear}}{\text{spindle gear}}.$$

With a 40-tooth gear on the spindle, $S$, and a 24-tooth gear on the worm, $W$, and two intermediate gears to complete the train, the spindle will be indexed $\frac{1}{250}$ revolution when the crank is moved 3 holes in the 18-hole circle.

The formulas given below can be used to obtain the same results:

$S$ = the number of teeth in the spindle gear.
$W$ = the number of teeth in the worm gear.
$g_1$ = the first intermediate gear on the idler stud.
$g_2$ = the second intermediate gear on the idler stud.
$N$ = the number of divisions required.
$N'$ = the number of divisions selected that can be obtained by plain indexing.

$$\mathcal{Q}=\frac{40}{N'}$$

The formula, $\frac{S}{W}=40-(Q\times N)$, requires one intermediate gear and is used when $N$ is less than $N'$.

The formula, $\frac{S}{W}=(Q\times N)-40$, requires two intermediate gears and is used when $N$ is greater than $N'$.

Using the formula to find the change gears, when the required number of divisions is 250, we get:

$$\frac{S}{W}=(Q\times N)-40$$

$$\mathcal{Q}=\frac{40}{N'}=\frac{40}{240}$$

$$\frac{S}{W}=\left(\frac{40}{240}\times 250\right)-40$$

$$\frac{S}{W}=\left(\frac{1}{6}\times 250\right)-40$$

$$\frac{S}{W}=\frac{250}{6}-40$$

$$\frac{S}{W}=\frac{250}{6}-\frac{240}{6}=\frac{10}{6}=\frac{5}{3}$$

Multiplying both the numerator and denominator by 8 to get gears with numbers of teeth that are available, we have:

$$\frac{S}{W}=\frac{5\times 8}{3\times 8}=\frac{40}{24},\ \frac{\text{spindle gear}}{\text{worm gear}}$$

## COMPOUND GEARING

When the movement of the plate relative to the spindle cannot be obtained by simple gearing, we use compound gearing.

The formulas for compound gearing are given below:

$$\frac{S \times g_1}{W \times g_2} = 40 - (Q \times N)$$ Used when $N$ is less than $N'$ and requires one intermediate gear.

$$\frac{S \times g_1}{W \times g_2} = (Q \times N) - 40$$ Used when $N$ is greater than $N'$ and requires one intermediate gear.

Number of divisions required is 239.

$N = 239$, $N' = 240$, and $Q = \frac{40}{240}$

$$\frac{S \times g_1}{W \times g_2} = 40 - (Q \times N)$$

$$\frac{S \times g_1}{W \times g_2} = 40 - \left(\frac{40}{240} \times 239\right)$$

$$\frac{S \times g_1}{W \times g_2} = 40 - \left(\frac{1}{6} \times 239\right)$$

$$\frac{S \times g_1}{W \times g_2} = 40 - \frac{239}{6}$$

$$\frac{S \times g_1}{W \times g_2} = \frac{240}{6} - \frac{239}{6} = \frac{1}{6}$$

$$\frac{S \times g_1}{W \times g_2} = \frac{\frac{1}{2} \times 2}{1 \times 6} = \frac{\left(\frac{1}{2} \times 64\right)\left(2 \times 12\right)}{(1 \times 64)\ (6 \times 12)} = \frac{32 \times 24}{64 \times 72}$$

$$\frac{S \times g_1}{W \times g_2} = \frac{32 \times 24}{64 \times 72}$$

$S = 32$ teeth, $W = 64$ teeth, $g_1 = 24$ teeth, $g_2 = 72$ teeth.

Fig. 61 shows the proper arrangement of the gears on the index head. The intermediate gear may have any number of teeth.

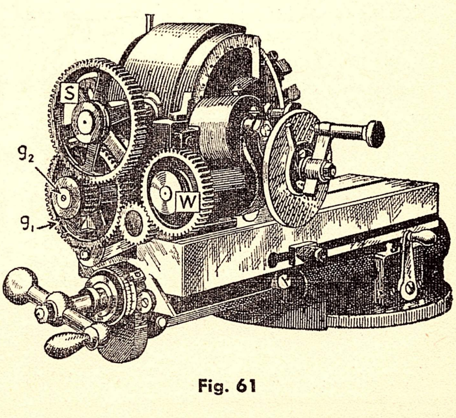


Fig. 61

### TEST YOUR ABILITY TO CALCULATE CHANGE GEARS

**34** Select the plate and circle, and calculate the change gears for indexing a gear with 119 teeth.

**35** It is required to cut a gear with 251 teeth. Find the plate and circle, and calculate the change gears for indexing, using compound gearing.

# Solutions to Questions and Exercises in Issue 9

## DIFFERENTIAL EQUATIONS
### DISINTEGRATION

**1** $M_t$ = amount remaining at any time, $t$
$M_0$ = original amount
$dM = -5\times10^{-3}M dt$;
$\log M = -5\times10^{-3}t + c$
when $t=0$ and $M=M_0$, $c=\log M_0$
$\log \dfrac{M}{M_0} = -5\times10^{-3}t$

| | *Part remaining* |
|---|---|
| $t=5$ days | 0.9753 |
| $t=100$ days | 0.6066 |
| $t=365$ days | 0.1612 |

$t_{\frac{1}{2}}=138.62$ days; $t_{\frac{1}{4}}=277.26$ days

**2** $\dfrac{dM}{dt} = -kM$; $\log M = -kt + c$
when $t=0$ and $M=M_0$, $c=\log M_0$
$\log \dfrac{M}{M_0} = -kt$; $\dfrac{M_t}{M_0} = e^{-kt}$; $M_t = M_0\, e^{-kt}$
$\log \frac{1}{2} = -kt_{\frac{1}{2}}$; $\log 2 = kt_{\frac{1}{2}}$; $t_{\frac{1}{2}} = \dfrac{\log 2}{k} = \dfrac{0.6931}{k}$

**3** (from **2**) $6.7 = \dfrac{0.6931}{k}$

$k = \dfrac{0.6931}{6.7}$; $M = 2e^{-.1034t}$

### SIMPLE DIFFERENTIAL EQUATIONS

**4** $\dfrac{dy}{dx} = \dfrac{y^3}{x^3}$; $\dfrac{dy}{x^3} - \dfrac{dx}{x^3} = 0$
$y^{-2} - x^{-2} = c$; $x^2 - y^2 = cx^2y^2$

**5** $\dfrac{dy}{dx} = e^{\frac{y}{x}} + \dfrac{y}{x}$. Let $\dfrac{y}{x} = u$; $\dfrac{dy}{dx} = x\dfrac{du}{dx} + u$
$x\dfrac{du}{dx} + u = e^u + u$; $\dfrac{du}{e^u} = \dfrac{dx}{x}$

$-e^{-4} = \log x + \log c$; $\log cx = -e^{-\frac{y}{x}}$
(This answer may appear in several different forms.)

**6** $\dfrac{dy}{dx} = \dfrac{y^2}{x^2}$; $\dfrac{dy}{x^2} = \dfrac{dx}{x^2}$
$-\dfrac{1}{y} + \dfrac{1}{x} = c$ or $\dfrac{1}{y} - \dfrac{1}{x} = c$; $x - y = cxy$

### LINEAR DIFFERENTIAL EQUATIONS

**7** $u(t)_1[Lv'(t) + R(v)t] + Rv(t)u'(t) = E\cos\alpha t$
Letting $Lv'(t) + Rv(t)$ equal 0, a solution is $v(t) = Ce^{-\frac{R}{L}}$; substituting in the first equation, $RCe^{-\frac{R}{L}t}u'(t) = E \cos\alpha t$ or

$$u'(t) = \frac{E\cos\alpha\, te^{\frac{R}{L}t}}{C} \text{ or } u(t) = \frac{E}{C}\int e^{\frac{R}{L}t}\cos\alpha t$$

$+D$; calling $\dfrac{R}{L} = k$, $u(t) = \dfrac{E}{C}\int e^{kt}\cos\alpha t$ $+$const.

$$= \frac{E}{C}\,\frac{e^{kt}(k\cos\alpha t + \alpha\sin\alpha t)}{\alpha^2 + k^2} + \text{const.}$$

$$= \frac{E}{C}\,\frac{e^{\frac{R}{L}t}\left(\frac{R}{L}\cos\alpha t + \alpha\sin\alpha t\right)}{\alpha^2 + \frac{R^2}{L^2}} + \text{const.}$$

$\therefore\ i = u(t)\cdot v(t)$

$$= CE^{-\frac{R}{L}t}\left[\frac{E}{C}\,\frac{e^{\frac{R}{L}t}\left(\frac{R}{L}\cos\alpha t + \alpha\sin\alpha t\right)}{\alpha^2 + \frac{R^2}{L^2}}\right] + D$$

$$= \frac{E\left(\frac{R}{L}\cos\alpha t + \alpha\sin\alpha t\right)}{\alpha^2 + \frac{R^2}{L^2}} + Ce^{-\frac{R}{L}t}$$

**8** $y' + ay + bx + c = 0$; let $ay \times bx + c = v$;
$a\dfrac{dy}{dx} + b = \dfrac{dv}{dx}$; $\dfrac{dy}{dx} = \dfrac{1}{a}\left(\dfrac{dv}{dx} - b\right)$;
$\dfrac{1}{a}\left(\dfrac{dv}{dx} - b\right) + v = 0$
$\dfrac{dv}{dx} - b + av = 0$; $\dfrac{dv}{dx} = b - av$, $\dfrac{dv}{b - av} = dx$
$-\dfrac{1}{a}\log(b - av) = x - \dfrac{1}{a}\log k$
$\log\dfrac{b - av}{k} = -ax$; $b - av = kc^{-ax}$
$b - a(ay + bx + c) = ke^{-ax}$

### APPROXIMATE SOLUTIONS

**9** $y'(x) = \frac{1}{2}y(x)$; when $x=2$, $y(2)=1$
when $x=2.5$, to find $y(2.5)$:
$y'(2) = \frac{1}{2}(1) = .5$; $y'(2.1) = .525$
$y'(2.2) = .551$; $y'(2.3) = .5788$
$y'(2.4) = .608$; $y'(2.5) = .638$
calculated value for $y(2.5) = 1.31$

### DIFFERENTIAL EQUATIONS OF SECOND ORDER

**10** $y'^2 + by = 0$ *Equation*
*Solution*: $y = A\cos\sqrt{b}t + B\sin\sqrt{b}t$
when $t=0$, $y=y_0$ and $y'=0$; $\therefore\ y_0 = A$
or $y = y_0\cos\sqrt{b}t + B\sin\sqrt{b}t$
$\left.\dfrac{dy}{dt}\right]_{t=0} = y' = 0 = -\sqrt{b}y_0\sin\sqrt{b}t + B\sqrt{b}\cos\sqrt{b}t$
or $B=0$ $\therefore$ equation $y = y_0\cos\sqrt{b}t$

## DIFFERENTIAL EQUATIONS
(continued)

**Application**

**11** This assumes a charged condenser; there is no externally impressed E.M.F.

In this case $\frac{d^2i}{dt^2}+\frac{1}{LC}=0$. Assume $R=0$.

$$Q=A\cos\sqrt{\frac{1}{LC}}t+B\sin\sqrt{\frac{1}{LC}}t$$

$$\frac{dQ}{dt}=i=-\frac{A}{\sqrt{LC}}\sin\frac{t}{\sqrt{LC}}+$$

$$\frac{B}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}}$$

if at $t=0$, $L=0$, then $B=0$

$$\therefore i=-\frac{A}{\sqrt{LC}}\sin\frac{t}{\sqrt{LC}}$$

Differentiation:

$\frac{di}{dt}=-\frac{A}{LC}\cos\frac{t}{\sqrt{LC}}$. Multiplying by $L$,

$$L\frac{di}{dt}=-\frac{A}{C}\cos\frac{t}{\sqrt{LC}}$$

If the condenser is initially charged with a potential $E_0$,

$$L\left(\frac{di}{dt}\right)_{t=0}=E_0=-\frac{A}{C};\ A=-CE_0$$

$$\therefore i=\frac{CE_0}{\sqrt{LC}}\sin\frac{t}{\sqrt{LC}}=$$

$$\frac{5\cdot10^{-6}E_0}{5\cdot10^{-3}}\sin\frac{t}{5\cdot10^{-3}}$$

$$=10^{-3}E_0\sin\frac{t}{5\cdot10^{-3}}$$

## MENSURATION

**1** 1×0.1009″ 1×0.1600″
1×0.1070″ 1×0.1000″

**2** 0.9297 1×0.1200″
1×0.1007″ 1×0.1000″
1×0.1090″ 1×0.5000″

**3** 9.3125″ 1×0.1100″
1×0.1005″ 1×2.000 ″
1×0.1020″ 1×3.000 ″
1×4.000 ″

**4** 39.37″

**5** 2.013 mph

**6** 981.5 cm./sec.$^2$

**7** (a) 144.9′
(b) 64.4′
(c) 16.1′

**8** 2,898.6 ft./sec.

**9** sp.g. = 7.092

**10** (a) vol. = 0.226 ft.$^3$
(b) wt. = 442.5 lb./ft.$^3$

**11** 14.24 lb./in.$^2$

**12** (a) 0.314 lb./in.$^2$
(b) 15.0 lb./in.$^2$
(c) Pressure in tunnel sub-atmospheric

**13** 92,750 in.−lb./piston

**14** (a) 776 H.P.
(b) Overcoming air resistance

**15** (a) 1,900 H.P.
(b) 1,417 Kw.

**16** (a) 61 H.P.
(b) 33,550 ft./sec.

## ANSWERS TO PUZZLE-PROBLEMS IN ISSUE NUMBER EIGHT

**52** Let $l$=length of stick, $h$=ht. of room, $d$=diameter of shaft, $\alpha=\angle$ stick makes with floor.

$$l=\frac{h}{\sin\alpha}+\frac{d\sin\alpha}{\cos^2\alpha} \qquad a$$

$$\frac{dl}{d\alpha}=-\frac{h\cos\alpha}{\sin^2\alpha}+\frac{d\cos\alpha}{\sin^2\alpha}$$

If $\frac{dl}{d\alpha}=0$, $\tan^3\alpha=\frac{h}{d}$, $\tan\alpha=\left(\frac{h}{d}\right)^{\frac{1}{3}}$

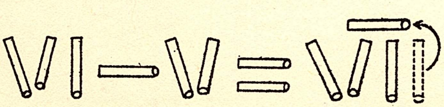
Substituting in a,

$$l=\left(h^{\frac{2}{3}}+d^{\frac{2}{3}}\right)^{\frac{3}{2}}=\left(18^{\frac{2}{3}}+10^{\frac{2}{3}}\right)^{\frac{3}{2}}=39'$$

(by slide rule)

**53** 4,004.996 ft.

**55** $6-5=\sqrt{1}$



**54** $\frac{DE}{CE}=\frac{DF}{14} \qquad a$

(Each side of the equation expresses the same trigonometric function of the same angle.)

$$DE=\frac{DF\cdot CE}{14} \qquad b$$

and $\frac{EF}{CE}=\frac{DF}{10} \qquad c$

$$EF=\frac{DF\cdot CE}{10} \qquad d$$

Add b and d

$$DE+EF=\frac{DF\cdot CE}{14}+\frac{DF\cdot CE}{10} \qquad e$$

But, $DE+EF=DF$

Divide e by $DF$ and you will get

$$1=\frac{CE}{14}+\frac{CE}{10} \text{ or } 70=12\ CE$$

and $CE$ = 5 feet 10 inches

## TABLE LII

### *BEHAVIOR OF MATERIALS OF CONSTRUCTION*

| | Elastic Limit (lbs. per sq. in.) | Tensile or compressive strength (lbs. per sq. in.) | Modulus of elasticity (stiffness) (lbs. per sq. in.) | Per cent elongation | Coefficient of linear expansion |
|---|---|---|---|---|---|
| **Structural steel** | 35,000 tension | 60,000 tension | 30,000,000 | 0.30 | 0.0000065 |
| **Nickel-chrome-SAE 3130 steel** | 215,000 tension | 250,000 tension | 30,000,000 | 0.10 | 0.0000065 |
| **Aluminum 17S-T** | 27,000 tension | 60,000 tension | 10,000,000 | 0.22 | 0.0000125 |
| **Concrete** (Average) | 2,000 compression | 4,000 compression | 4,000,000 | 0.002 | 0.0000065 |
| **Douglas Fir** (Dry) **Parallel to grain** | 6,500 compression | 7,400 compression | 1,400,000 | 0.008 | ——— |

All steels have properties in compression similar to those in tension.

Concrete has very low tensile properties, so it is never used in tension.

The properties of wood vary with the moisture content and the direction of the grain.

## TABLE LIII

### GUIDE TO PROPER CUTTING SPEEDS—(Feet per minute)

| Material in Cutter | Hard Steel | Medium Steel | Soft Steel | Malleable Iron | Chilled Cast Iron | Hard Cast Iron | Soft Cast Iron | Hard Bronze | Bronze | Brass | Aluminum |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | MATERIAL TO BE CUT | | | | | | | | | | |
| **Carbon steel** | | 30 to 40 | 30 to 45 | 35 to 50 | | | 30 to 40 | | 30 to 60 | 40 to 80 | 250 to 500 |
| **High speed steel** | 30 to 50 | 50 to 80 | 60 to 90 | 70 to 100 | | 30 to 50 | 50 to 80 | 30 to 50 | 65 to 130 | 70 to 175 | 500 to 1000 |
| **Super high speed steel** | 40 to 70 | 60 to 90 | 70 to 100 | 80 to 125 | 30 to 50 | 40 to 70 | 60 to 115 | 50 to 70 | | | |
| **Stellite** | | | | 115 to 150 | 40 to 60 | 60 to 90 | 90 to 130 | | 100 to 160 | 150 to 250 | 800 te 1500 |
| **Cemented carbide** | 100 to 200 | 125 to 300 | 150 to 400 | 250 to 400 | 100 to 250 | 150 to 300 | 250 to 350 | 125 to 250 | 200 to 500 | 350 to 700 | 1000 to 2000 |

## TABLE LIV

### CONE HEIGHT CALCULATIONS

| Diameter of Drill $d$ | Cone Height $h$ | Diameter of Drill $d$ | Cone Height $h$ | Diameter of Drill $d$ | Cone Height $h$ | Diameter of Drill $d$ | Cone Height $h$ |
|---|---|---|---|---|---|---|---|
| $\frac{1}{4}$ | 0.075 | $\frac{3}{4}$ | 0.225 | $1\frac{1}{8}$ | 0.338 | $1\frac{1}{2}$ | 0.451 |
| $\frac{3}{8}$ | 0.113 | $\frac{7}{8}$ | 0.263 | $1\frac{1}{4}$ | 0.376 | $1\frac{5}{8}$ | 0.488 |
| $\frac{1}{2}$ | 0.150 | 1 | 0.301 | $1\frac{3}{8}$ | 0.414 | $1\frac{3}{4}$ | 0.526 |
| $\frac{5}{8}$ | 0.188 | | | | | 2 | 0.601 |

## TABLE LV
### *STANDARD THREAD SYSTEMS*

| National Coarse-Thread Series | | | | National Fine-Thread Series | | | |
|---|---|---|---|---|---|---|---|
| Sizes | Threads per Inch $n$ | Major Diameter $D$ (Inches) | Depth of Thread $h$ (Inches) | Sizes | Threads per Inch $n$ | Major Diameter $D$ (Inches) | Depth of Thread $h$ (Inches) |
| 1 | 64 | 0.073 | 0.01015 | 0 | 80 | 0.060 | 0.00812 |
| 2 | 56 | .086 | .01160 | 1 | 72 | .073 | .00902 |
| 3 | 48 | .099 | .01353 | 2 | 64 | .086 | .01015 |
| 4 | 40 | .112 | .01624 | 3 | 56 | .099 | .01160 |
| 5 | 40 | .125 | .01624 | 4 | 48 | .112 | .01353 |
| 6 | 32 | .138 | .02030 | 5 | 44 | .125 | .01476 |
| 8 | 32 | .164 | .02030 | 6 | 40 | .138 | .01624 |
| 10 | 24 | .190 | .02706 | 8 | 36 | .164 | .01804 |
| 12 | 24 | .216 | .02706 | 10 | 32 | .190 | .02030 |
| $\frac{1}{4}$ | 20 | .2500 | .03248 | 12 | 28 | .216 | .02320 |
| $\frac{5}{16}$ | 18 | .3125 | .03608 | $\frac{1}{4}$ | 28 | .2500 | .02320 |
| $\frac{3}{8}$ | 16 | .3750 | .04059 | $\frac{5}{16}$ | 24 | .3125 | .02706 |
| $\frac{7}{16}$ | 14 | .4375 | .04639 | $\frac{3}{8}$ | 24 | .3750 | .02706 |
| $\frac{1}{2}$ | 13 | .5000 | .04996 | $\frac{7}{16}$ | 20 | .4375 | .02348 |
| $\frac{9}{16}$ | 12 | .5625 | .05413 | $\frac{1}{2}$ | 20 | .5000 | .03248 |
| $\frac{5}{8}$ | 11 | .6250 | .05905 | $\frac{9}{16}$ | 18 | .5625 | .03608 |
| $\frac{3}{4}$ | 10 | .7500 | .06495 | $\frac{5}{8}$ | 18 | .6250 | .03608 |
| $\frac{7}{8}$ | 9 | .8750 | .07217 | $\frac{3}{4}$ | 16 | .7500 | .04059 |
| 1 | 8 | 1.0000 | .08119 | $\frac{7}{8}$ | 14 | .8750 | .04639 |
| $1\frac{1}{8}$ | 7 | 1.1250 | .09279 | 1 | 14 | 1.0000 | .04639 |
| $1\frac{1}{4}$ | 7 | 1.2500 | .09279 | $1\frac{1}{8}$ | 12 | 1.1250 | .05413 |
| $1\frac{1}{2}$ | 6 | 1.5000 | .10825 | $1\frac{1}{4}$ | 12 | 1.2500 | .05413 |
| $1\frac{3}{4}$ | 5 | 1.7500 | .12990 | $1\frac{1}{2}$ | 12 | 1.5000 | .05413 |
| 2 | $4\frac{1}{2}$ | 2.0000 | .14434 | | | | |
| $2\frac{1}{4}$ | $4\frac{1}{2}$ | 2.2500 | .14434 | | | | |
| $2\frac{1}{2}$ | 4 | 2.5000 | .16238 | | | | |
| $2\frac{3}{4}$ | 4 | 2.7500 | .16238 | | | | |
| 3 | 4 | 3.0000 | .16238 | | | | |